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THE LATENT HEAT OF RECALESCENCE IN IRON AND STEEL.

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THE phenomenon of recalescence in iron and steel has, in recent years, been the subject of considerable study. Apparently, all the properties of iron and steel undergo a change, greater or less, at the temperature at which the magnetic properties are lost. Such a change of properties indicates what is commonly termed a change of state, that is, a change of the physical state or constitution of the substance. As is usual with such changes of state, this change in iron and steel is accompanied by an absorption of heat when passing from the colder to the warmer state. As this heat is liberated at the corresponding change on cooling, and as it seems to be used solely to produce the change, it may correctly be termed latent heat. If the name of the most striking phenomenon connected with the change, that of recalescence, be given to the whole phenomenon, then we may correctly speak of the latent heat of recalescence. Strict analogy, however, would require a name for the phenomenon during rise of temperature, just as we speak of latent heat of fusion or of vaporization. Or, a name might be given to the substance in the heated state, and we should speak of the latent heat of steel in that state as we speak of the latent heat of water or of steam.

The present work was undertaken with a view to determine the value of this latent heat, and to learn in what way, if in any, it depended upon the composition.

Hopkinson[1] has shown that a piece of iron or steel, if heated above the point of recalescence and allowed to cool slowly, pauses in decline of temperature, for several seconds, at the temperature where it becomes magnetic. He also showed graphically that the amount of heat given off in this interval is 175 to 200 times that given off by the same piece of matter, in a fall of temperature of one degree, just above or below this temperature. That is, the piece under observation would give out heat without fall of temperature, for a period long enough to reduce its temperature 175 or 200 degrees if the cooling were perfectly uniform.

These figures, however, give the latent heat in terms of the specific heat of iron at this temperature. The best determinations of this quantity have been made by Pionchon,[2] and more recently by Harker,[3] who has corrected the values of the former, besides making a re-determination. Their figure for the specific heat of iron just above or below this temperature is .20 times that of water. Multiplying by this 175 or 200 degrees, we get 35 or 40 calories per gram, as the latent heat of recalescence in the specimen examined. The very magnitude of this quantity invites further study. For, comparing with other substances, we find it relatively greater than even the latent heat of the ice-water change, which is only about 160 ice gram degrees, or 80 water gram degrees.

The specific heat of a substance is determined from the total quantity of heat given up by a unit of mass in cooling from a given high temperature to 0° C., or to some ordinary temperature. If the upper temperature is above that of the change of state, then the latent heat will be included with the rest. If the heat be expressed by a formula, as is done by Pionchon, the latent heat should be separated from the other and appear as a constant term, in formulas applying to temperatures above the change of state. Pionchon gives several formulas for the total heat between 0° C. and various high temperatures. They contain a constant term, but it is much larger than the value he gives for the latent heat, and it also appears with

[1] Phil. Trans., 1889, A, pp. 463–465; Proc. Roy. Soc., Vol. 48, p. 442; or Original Papers, Vol. II., Nos. 30, 31, pp. 217–226.

[2] Ann. d. Chim. e. Phys. (6), Vol. 11, p. 33, 1887.

[3] Phil. Mag., Vol. 10, p. 430, 1905.

a negative sign. He finds for the latent heat 5.3 calories.[1] Harker does not give a value for the latent heat, but by plotting his figures[2] one finds the figures 7.5 calories.

Hopkinson has also shown[3] that the phenomenon of recalescence is greatly affected by the composition of the steel. This fact and the wide differences of previous results were the incentive for the present work. The object of the work is, therefore, to determine the specific heat and latent heat of several samples of steel and study the results in connection with analyses of the studied samples. Both the calorimetric and the rate of cooling methods have been used. The first part of the research comprised the calorimetric work. Because of certain features of the apparatus, and because the main object of the work was the value of the latent heat, the range of temperature has not been great, *viz.*, 460° to 860 C°.

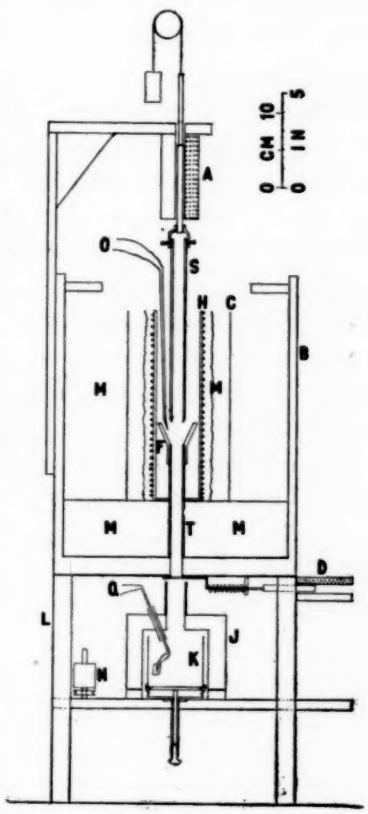


Fig. 1.

The Heating Apparatus. — This was an electric resistance furnace constructed by the writer as shown in Fig. 1.

A square wooden box, *B*, was made, whose inside measure was 30 cm. side and 38 cm. deep. It was open at the top. Through the center of the bottom was a hole 2 cm. in diameter. A porcelain tube, *T*, 15 cm. long, and having an internal diameter of 1.5 cm., was placed vertically over this hole. The whole bottom of the box, around the tube, was then covered to a depth of 8 cm. with

[1] Loc. cit , pp. 77, 106.
[2] Loc. cit., p. 437.
[3] Loc. cit., p. 463.

magnesia, *M*, firmly packed. On top of this was placed a sheet of asbestos board on which was stood a sheet iron cylinder, *C*, 18 or 20 cm. in diameter, and 25 cm. high. (A piece of stove pipe would be suitable.) This cylinder was placed with its axis coinciding with that of the box. The space between the cylinder and the box was also tightly packed with magnesia as far up as the top of the cylinder. The object of this cylinder was to allow the heating coil to be removed without disturbing the packing. The heating coil, *H*, was placed in the center of the cylinder. The heating coil was a clay cylinder wound with nickel wire. Two battery porous cups were used, placed end to end. The bottom of one cup was cut off and the end ground to fit neatly the other cup. The bottom of the second cup was pierced with a hole large enough to admit the tube, *T*. The cylinders were wound separately. Nickel wire No. 20 B. & S. gauge was used. The cups were first wrapped with two layers of asbestos paper, then wound with wire, and then covered with a paste made of magnesia and water. The wire, after being annealed, was wound on by hand, the turns being about 3 mm. apart. The paste was spread on smoothly to a thickness of half a centimeter, the ends of the wires, in the meantime, being held by clamps. When the paste had dried, the coils were placed in the center of the space within the cylinder, *C*, the tube, *T*, projecting up into the lower cup some 8 cm. The lead wires were brought to the top of the box through glass tubes, and the remaining space between the coil, *H*, and cylinder, *C*, was tightly packed with dry magnesia. A layer about 2 cm. thick of the magnesia paste, nearly dry, was then spread over the top and firmly pounded down, so that the top was even and hard. The shell of magnesia on the heating coils held the wires in place and remained intact even after many heatings. When, after long use, the wires had to be renewed and the clay cups were withdrawn, this shell came out whole and was firm enough to require a metal tool for its removal. The heating coil having two parts was a decided advantage, for it not only allowed greater variation in heating, but also made the process of packing easier. Moreover, if one coil were destroyed, it alone need be rewound. This completed the furnace, it consisting essentially of a clay cylinder wound with wire, and

surrounded with a thick packing of magnesia, there being a tube connecting the space inside the cylinder with the space outside beneath the furnace.

The furnace constructed in this manner proved both durable and efficient. Temperatures above 1,000° C. were not attempted, but the furnace was heated many times to somewhat lower temperatures, 700° to 900° C., the period of the high temperature being four or five hours. Even when the temperature had been maintained near to 1,000° C. for several hours, the outside of the box could be touched with the hand with no discomfort. The inside of the box may have been as warm as 100° C.; hence, the magnesia would maintain a gradient of 80° C. per cm. The heating current was alternating current taken from the lighting circuit at 110 volts. The current was regulated with a rheostat. To maintain a temperature of 1,000° C. between 5 and 6 amperes were required. The resistance at 1,000° was about 10 ohms; hence, the energy radiated by the furnace when the interior was at 1,000°, must have been about 360 watts.

Calorimetry. — Two methods of calorimetry were tried. The first method was that of Hesehus, successfully used and described by Waterman.[1] While the method will give accurate results if sufficient care is taken, it has still one serious defect in that two temperatures must be measured with great precision on two different thermometers. Even if these are carefully calibrated and compared, an extra source of error is introduced at a vital part of the process. In addition to this, the increased difficulty of manipulation is an objection to the method. The results not being sufficiently accurate, the method was abandoned and the ordinary method of mixtures used.

The furnace was mounted on four legs, *L*, 30 cm. long. Connecting the legs on each side were cross sticks on which was placed a board 65 cm. long and 25 cm. wide. On this board was placed a small turntable consisting of a disk of hard rubber, 7 cm. in diameter, grooved on the edge, and supported by its center on the upper end of a small shaft. The lower end of the shaft rested on the conical end of a screw, by which the height of the disk could

[1] PHYSICAL REVIEW, Vol. 4, p. 161, 1897.

be adjusted. A small electric motor, *N*, was also mounted on the board and connected to the turntable by a small belt.

The calorimeter, *K*, was a polished silver cup 7 cm. high and 7 cm. in diameter, and had a weight of 50 grams. Its water equivalent was, therefore, 2.8 grams. The calorimeter was carried on the turntable, resting on three bits of cork. The turntable was so placed that when the board was pushed in to a stop, the calorimeter was directly under the center of the furnace (this is the position shown in the figure), and when drawn out to another stop, the calorimeter was in a convenient position to remove and change. The wires leading to the motor were so arranged that the circuit closed automatically when the board was pushed in.

To reduce radiation, a water jacket, *J*, was made, which completely covered the calorimeter except for a circle 2.5 cm. in diameter above the center. The water jacket was bright on the inside and had a water thickness of 2 cm. The outside of the jacket was covered with two thicknesses of asbestos paper. Through the top of the jacket passed two tubes. One of these was 2.5 cm. in diameter and was in the center of the top. This opening served to admit the hot body to the calorimeter and was lined on the inside with asbestos paper, the tube of lining extending above the top of the jacket so that it came within .5 cm. of the bottom of the furnace. The other hole had a diameter of 1 cm. and had an oblique position. The purpose of this tube was to admit the thermometer. The thermometer, *Q*, consisted of a thermopile of four elements of copper-german silver. The wires in the elements were 60 cm. long, except the terminal copper wires, which were long enough to reach the galvanometer. The pile was connected directly to a d'Arsonval galvanometer having a resistance of 114 ohms. The pile was carefully calibrated with mercury thermometers and found to give a deflection of 23.3 mm. (scale distance, 50 cm.) per degree Centigrade difference in temperature of the junctions. The deflection was found to be a linear function of the difference in temperature, which fact permitted the rise in temperature in the calorimeter to be derived directly from the difference in deflection, without first reducing to initial and final temperatures. The constant temperature junctions were fixed into a glass tube containing oil, and this tube

placed in water contained in a jar insulated with a thick covering of magnesia. The temperature of this bath was determined to a tenth of a degree by a mercury thermometer. This temperature was very constant, not changing by a degree in several hours. Since, in any one operation, the pile was in use only two or three minutes, the constant temperature ends could not have changed by one one-hundredth degree during the operation. The wires of the pile were bound together in a rubber tube and one end passed through the small hole in the water jacket. It was securely fastened in this hole with the ends projecting inward in such a position that when the jacket was placed over the calorimeter, the ends dipped therein. The ends were spread apart and small pieces of mica (1 cm. square) attached to them. The ends and mica were covered with collodion to give insulation and also to hold on the mica vanes. These vanes served to stir the water when the calorimeter was rotated. This method of stirring proved very effective and convenient. (If the axis of the calorimeter were made to describe a cone, the water could be rotated without rotating the calorimeter. This might be desirable in some cases.)

Manipulation. — The specimen whose specific heat was to be determined was made into a sphere small enough to pass through the tube, *T*. This tube projected some 8 cm. into the chamber of the furnace, and carried on its upper end a small funnel, *F*, made of cast iron. (For temperatures above 800° the iron funnel is unsatisfactory by reason of its oxidation, the oxide sticking to the rod, *S*, and preventing its rising. A clay funnel would, no doubt, solve the difficulty.) The funnel was closed by a rod of clay or porcelain, *S*, larger than the hole, resting in it. This rod extended through the cover of the furnace and carried at its upper end a short iron rod which extended into a solenoid, *A*, rigidly fixed above the furnace. The rod could be lifted by passing a current through the solenoid. The iron rod was continued in a smaller brass rod, to upper end of which was attached a counterbalance. A notch in the brass rod, and a detent (not shown) served to hold up the rod when once lifted. This was to prevent the rod from falling back with a jar into the funnel. The lower end of the tube, *T*, was covered by a slide which could be drawn aside by a magnet, *D*,

and closed by a spring. The wires were so connected that the circuit through the upper solenoid was closed by the opening of the slide. This insured the funnel opening immediately after the slide.

When the specimen was introduced into the furnace, it rested in the funnel beside the rod, *S*, and when this was lifted, the ball rolled down the funnel and fell directly into the calorimeter. Its total fall was 28 cm., of which 10 cm. was through hot tube in the furnace, 8 cm. through tube varying from the furnace temperature to atmospheric temperature, and the remainder, 10 cm., through the water jacket at atmospheric temperature. There was, therefore, very little opportunity for cooling before the calorimeter was reached.

The size of the specimen and of the calorimeter was so proportioned that the rise in temperature of the latter was small. This reduced the radiation to negligible proportions. The calorimeter temperature would rise quickly and remain stationary for a much longer period. Moreover, the initial temperature of the calorimeter could be varied several degrees without appreciable change in the results. The greatest error observed which could be assigned to this cause, was about two per cent., and this was a very unfavorable case. The temperature of the calorimeter was usually such that the final temperature was not more than one or two degrees above that of the water jacket. (The errors due to radiation might be wholly eliminated if the water in the jacket were heated at the same time and to the same degree as that in the calorimeter. By making the chamber of the furnace double and providing a piece whose heat capacity should bear the same ratio to that of the calorimetric piece that the heat capacity of the jacket bears to that of the calorimeter, and then dropping the two pieces at once into the jacket and calorimeter respectively, with proper stirring, the desired result should be obtained. Or the jacket might be heated electrically and equality of temperature maintained by the use of a second thermo-couple.)

The temperature of the furnace was determined by a platinum-platinum-rhodium couple, *O*. The E.M.F. was measured on a Wolff potentiometer, adjusted to read directly in hundredths of a millivolt. The temperature of the hot junction was then found from a table made for the purpose. The element was compared with a standard element made of wires from the same pieces and calibrated

at the National Bureau of Standards in Washington, District of Columbia. The formula given for the standard element was:

$$E = -0.280 + 0.00788t + 0.0000182t^2.$$

The element used in these experiments had a slightly higher E.M.F. and the values for temperature and E.M.F. in the table used were as follows, the cold junction being at 0° C.

Temp.	300	400	500	600	700	800	900	1,000	degrees C.
E.M.F.	2.14	3.08	4.06	5.05	6.08	7.14	8.24	9.37	millivolts.

The cold junctions were fixed in a glass tube in a manner quite similar to that in case of the pile Q, and they were kept in the same water bath. The hot junction was inserted in the furnace bare, the wires passing through a length of two-way clay tubing. The junction was very close to the funnel and could almost be touched by the ball. In some later experiments the junction was broken and the wires reunited. This new junction was then compared with the standard and found to agree exactly with the former calibration. The furnace temperature rose slowly during any series of experiments (see below), but the rate was so slow that it was assumed that the ball and element were at the same temperature. This assumption was justified by the fact that when the ball had been in the furnace for fifteen minutes it became perfectly indistinguishable from its surroundings. This was repeatedly observed. A difference of a single degree would have rendered it visible. The ball was left in the furnace, as a rule for twenty minutes.

A single experiment comprised the following operations: (1) Placing specimen in furnace, (2) filling and weighing calorimeter, (3) placing same in jacket under furnace, (4) setting and reading potentiometer, (5) reading galvanometer for initial temperature of calorimeter, (6) dropping specimen into calorimeter, (7) reading galvanometer for final temperature of calorimeter, (8) reading thermometer in water bath, (9) removing calorimeter and specimen.

The time between operations 3 and 4 was sufficient to reduce the previous set of readings. Below is given a specimen set of observations as taken from the note book.

Specimen No. 7.

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
8	227.2 grams.	650°	400 *R*	482 *B*	882	16.2°	748°	682.2	7.9
	180.0 "	740°		2.1°	3.79°	18.3°	730°	.1513	

Weight of ball, 6.103 grams; weight of covering, 0.327 gram.
Heat capacity of covering per 100°, 1.09 calories.

Headings of columns, first row.	Second row, calculations.
A. Number of experiment.	
B. Weight of water and calorimeter (obs.).	Water equivalent of same.
C. Reading of potentiometer (obs.).	Temperature corresponding in table.
D. Reading of galvanometer before drop (obs.).	
E. Reading of galvanometer after drop (obs.).	Diff. temperature bet. cal. & bath.
F. Swing of galvanometer $= E - D$ (calc.).	Rise in cal. temp.
G. Temperature of water bath (obs.).	Final temp. of cal. $= G + E_1$.
H. True temp. of furnace $= C_1 + \frac{G}{2}$.	Fall in temp. of specimen $= H - G_1$.
I. Heat given to calorimeter $= B_1 F_1$.	Mean specific heat $= \frac{I - J}{6.103 H_1}$
J. Heat given to calorimeter by covering $= \frac{H_1 1.09}{100}$	

The first part of the research consisted in the determination of the specific heat of several samples of steel having different composition. In addition to this, experiments were made on platinum, nickel, aluminum and copper. The results of these experiments follow:

Temp.	Platinum.		Nickel.		Aluminum.		Copper.	
	Total.	Specific.	Total.	Specific.	Total.	Specific.	Total.	Specific.
400–20			42.4	.1115	86.6	.2280		
500–20			55.0	.1145	113.8	.2370		
550–20	17.6	.03331			128.0	.2415	51.6	.0975
600–20	19.4	.03344	68.0	.1173	142.7	.2460	57.2	.0987
700–20	22.9	.03370	81.6	.1200			68.7	.1010
800–20	26.4	.03395	95.7	.1227				
900–20	30.1	.03422						

Only a few experiments were made on aluminum and copper; hence the figures given have little value. The pieces were taken from commercial stock found in the shop of the laboratory. The platinum and nickel specimens were obtained from Baker & Com-

pany, of Newark, New Jersey. No analyses were made of these pieces. The figures given for nickel and platinum are derived from the means of about 40 experiments in each case.

The steel specimens are numbered consecutively from 1 to 9. Nos. 1 and 2 are from pieces of machinery steel or rolled iron found in the shop of the laboratory. No. 3 is a piece of Stubbs tool steel. Nos. 4 to 9 were supplied by the firm of Pratt & Inman, of Worcester, Massachusetts, from their regular stock. Nos. 4 to 7, inclusive, were made by the Gautier Department of the Cambria Steel Company, of Johnstown, Pennsylvania, and Nos. 8 and 9 were made by the Crucible Steel Company and the Colonial Steel Company, respectively.

All the samples were analyzed in the chemical laboratory of Clark University, by Mr. E. A. Harrington, a student in the university.

The results were as follows, in fractions of 1 per cent:

No.	*C*	*P*	*S*	Mn.
1	.09			
2	.05			
3	.97	.07	.01	
4	.08	.08	.01	.82
5	.18	.13	.03	.77
6	.26	.16	.01	.42
7	.28	.19	.04	.83
8	.40	.02	.02	.49
9	.79	.03	.02	.40

As already stated, the specimens were used in the shape of a ball. These balls were about 11 mm. in diameter and were covered with platinum foil. The balls weighed between 5 and 6 grams each. They were first given a thin nickel plating. Two circular disks of foil were cut, each large enough to cover more than half of the ball. These were annealed by heating to redness in a gas flame, and pressed into shape by placing them on a cake of wax and forcing the ball and foil into the wax. The two cups thus formed covered the ball and overlapped by one or two millimeters. The ball was then seized in tongs, in such a way as to press the two caps together, and heated to redness in a flame. By quickly hammering the joint, the two halves of the covering could be welded together. By re-

heating and rolling the ball between two smooth pieces of iron, the covering was made to fit the ball closely. No covering, however, remained intact through a whole series of experiments. It was usually punctured by the sixth or eighth heating. This gave no trouble, for the piece was frequently weighed and the change in weight was very small, even when quite a crack appeared. The foil used was 0.03 mm. thick.

Results. — The data obtained were reduced to mean specific heat for each experiment, and the results plotted on cross-section paper. The mean was then found by drawing a curve through the points plotted. Something over 30 points were found for each curve and nearly all were within one per cent. of the mean line. The final temperature of the calorimeter was always very near 20° C. and this temperature is taken as the lower temperature in subsequent calculations. The total heat per gram was then calculated from the mean specific heat curve. Although this is a round-about method, since the total heat is directly determined, it has some advantages in finding the mean. For, if $q_0{}^T$ be the total heat required to heat the piece from 0° to $T°$, and c the specific heat at any temperature, then

$$q_0{}^T = \int_0^T c\,dT$$

where c is a function of T. If c can be expressed as a very rapidly converging power series, as

$$c = \alpha + 2\beta T + 3\gamma T^2 + \cdots$$

where α, β, γ, etc., are constants, then

$$q_0{}^T = \alpha \int_0^T dT + \beta \int_0^T 2T\,dT + \gamma \int_0^T 3T^2 dT + \cdots$$

$$= \alpha T + \beta T^2 + \gamma T^3 + \cdots$$

and

$$\frac{q_0{}^T}{T} = \alpha + \beta T + \gamma T^2 + \cdots = S_0{}^T.$$

If γT^2, etc., are small in comparison with α and βT, as is usually the case, then S becomes a linear function of T.

If the latent heat L appears then the formulas become:

$$q_0^T = L + \int c dT = L + aT + \beta T^2 + \gamma T^3 + \cdots$$

and

$$S_0^T = \frac{L}{T} + a + \beta T + \gamma T^2 + \cdots$$

For the curve beyond the point where the latent heat appears, the latent heat should be subtracted before dividing by T, giving:

$$S = \frac{q_0^T - L}{T} = a + \beta T + \gamma T^2 + \cdots$$

TABLE I.

Total Heat $= q_{20}^T$.

Temp.	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.	No. 9.
470	57.6	55.0	54.8	51.8	57.5	56.0	55.3	55.6	55.0
520	64.8	62.6	63.2	59.3	65.0	63.5	63.0	63.6	63.0
570	72.4	70.5	72.3	67.1	72.6	71.1	71.3	72.0	71.3
620	80.8	78.6	81.7	75.9	80.7	79.6	80.2	80.6	80.2
670	90.8	87.1	91.4	86.0	90.1	89.3	89.9	89.7	89.6
720	103.6	97.5	101.6	97.0	101.0	99.5	99.8	99.4	99.5
770	117.7	110.8	126.5	111.1	118.2	117.0	119.0	125.4	124.9
820	128.4	120.8	134.8	127.1	129.6	128.5	128.0	134.6	134.9
860	136.1	128.5	139.8	135.2	136.2	134.0	134.1	142.0	143.0

TABLE II.

Mean Specific Heat $= S_{20}^T = \frac{q_{20}^T}{T - 20}$.

Temp.	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.	No. 9.
470	.1281	.1223	.1220	.1150	.1280	.1242	.1220	.1231	.1218
520	.1296	.1252	.1268	.1182	.1300	.1269	.1260	.1270	.1260
570	.1316	.1281	.1313	.1218	.1319	.1293	.1298	.1308	.1300
620	.1346	.1310	.1359	.1262	.1347	.1328	.1338	.1344	.1338
670	.1397	.1340	.1405	.1322	.1386	.1372	.1380	.1381	.1377
720	.1480	.1393	.1451	.1385	.1444	.1419	.1421	.1420	.1420
770	.1570	.1477	.1684	.1485	.1577	.1559	.1590	.1670	.1665
820	.1605	.1510	.1683	.1590	.1619	.1606	.1600	.1680	.1685
860	.1620	.1530	.1682	.1610	.1623	.1595	.1597	.1687	.1702

The values of q and S are given in Tables I. and II., respectively, and these values are plotted in Figs. 2 and 3. Plots are given only

for samples 3 to 9 inclusive. In the plots the abscissæ represent temperature and the ordinates, in Fig. 2, represent $q_{20}{}^{T}$ and in Fig. 3, $S_{20}{}^{T}$. For the sake of comparison, the curves are placed one

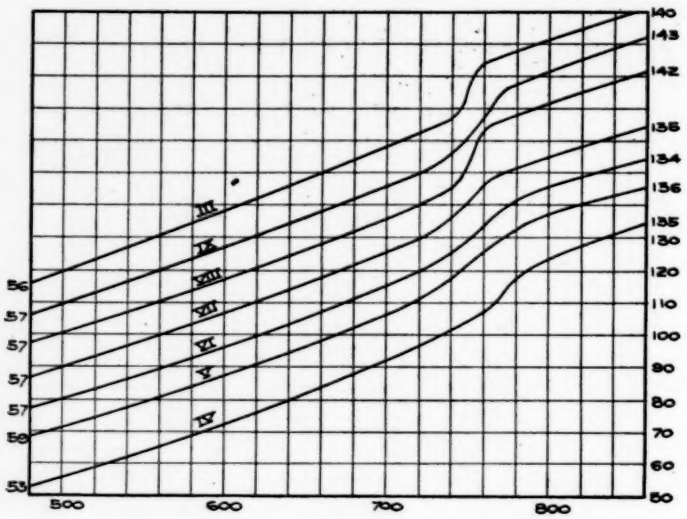


Fig. 2

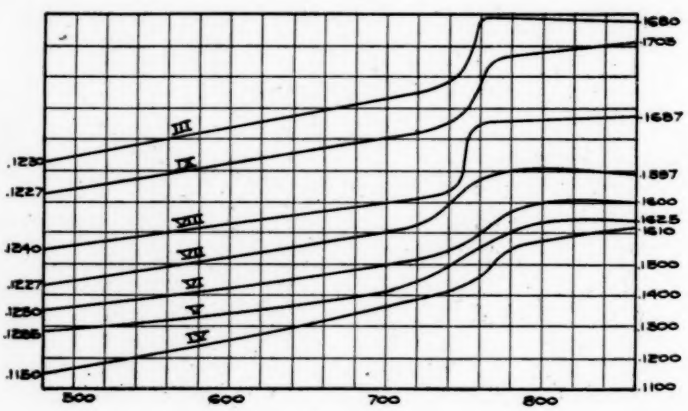


Fig. 3.

above another, the initial and final values being given at the left and right respectively. The scale of ordinates is also given at the right.

Inspection of the curves shows that the values of both q and S increase with the temperature. The curves for S between 500° and

700° are nearly straight and rise at a rate varying between .004 and .009 per 100°. This gives the value of β (γ and higher coefficients being neglected). The greatest difference between values of q for different samples are between 1 and 4 at 470°, 5.8 = 10 per cent.; between 3 and 4 at 620°, 5.8 = 6 per cent.; between 2 and 9 at 820°, 14.1 = 11 per cent.

Taking sample 7 for example, we find, between 470 and 720 β = .00008. Between 720 and 780 latent heat appears to the amount 9.7 cal. Subtracting this from values of q in the table, we have:

T	770	820	860
$Q_{20}{}^{T}$	108.3	118.3	124.4
$S_{20}{}^{T}$	.1444	.1479	.1481

These values are not sufficient to calculate β, but show that it may even become negative. The figures and the curve here indicate that it is very nearly zero. By extrapolation, we find $\alpha = S_{20} =$ .0860, whence

$$S_{20}{}^{T} = .0860 + .00008\ T$$

and

$Q_{20}{}^{T} = .0860\ T + .00008\ T^{2}$ between $T = 470°$ and $T = 720°$.

The value .0860 for α being too low, shows that another coefficient is needed. Above 820° the formulas become

$$S_{20}{}^{T} = \frac{9.7}{T} + .1480 \pm$$

and

$$Q_{20}{}^{T} = 9.7 + .1480\ T.$$

TABLE III.

Specific Heat = c.

Temp.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.	No. 9.
470–520	.150	.150	.150	.154	.160	.160
520–570	.156	.152	.152	.174	.168	.166
570–620	.176	.162	.170	.178	.172	.178
620–670	.182	.188	.194	.194	.182	.188
670–720	.220	.218	.204	.198	.194	.198
720–770	.282	.344	.350	.384	.520	.508
770–820	.320	.228	.230	.180	.184	.200
820–860	.202	.165	.137	.152	.185	.202

The specific heat c_T at the temperature T is given by the slope of the curve q. These can be calculated from Table I. and are given in Table III.

Table IV. gives the figures obtained for a single sample, No. 7, as recorded in the notebook.

TABLE IV.

No.	Temp., Deg. C.	Total Heat $= q^T_{20}$ Calories Per Gram.	Mean Sp. Heat $= S^T_{20}$
1	520	62.7	.1254
2	552	68.3	.1282
3	575	72.4	.1304
4	612	78.8	.1331
5	618	81.2	.1358
6	631	83.5	.1366
7	634	82.6	.1346
8	636	83.5	.1355
9	656	86.5	.1359
10	664	88.1	.1367
11	665	88.9	.1376
12	672	90.4	.1386
13	673	89.5	.1370
14	687	92.7	.1389
15	692	93.6	.1390
16	692	95.0	.1414
17	700	96.1	.1413
18	708	96.4	.1401
19	721	100.6	.1435
20	728	102.3	.1444
21	732	102.5	.1439
22	745	112.2	.1546
23	750	110.5	.1513
24	764	118.4	.1591
25	767	118.3	.1583
26	782	122.4	.1604
27	790	120.8	.1568
28	813	125.0	.1575
29	817	127.5	.1600
30	825	126.9	.1575

The second part of the research consisted in studying the manner in which the various kinds of steel increased or decreased in temperature. A cylinder of each of Nos. 4 to 9 was cut, having a length of 5.1 cm. and a diameter of 2 cm. A hole large enough

to admit the platinum-platinum-rhodium element was drilled in the axis to a depth of 2.5 cm. so that the end of the hole was near the center of the piece. These cylinders were covered with mica and one or two layers of asbestos paper, then wound with wire having a resistance of about 6 ohms. The whole was then thickly covered with a paste of magnesia wet with water, the thermo-element previously used in the furnace being inserted in the hole in the iron. The paste was dried and the whole thing packed in dry magnesia so that there was 5 to 8 cm. of magnesia between the cylinder and the open air. The cylinder was then heated by passing a current through the wire. Three amperes was sufficient to carry the piece slowly to a temperature of 850° C. In order to read the temperature directly on the galvanometer, a resistance was put in series with the galvanometer such that one millivolt on the potentiometer produced a deflection of 10 cm. on the scale of the galvanometer. The potentiometer was then set at 5.50 millivolts which corresponded to a temperature of the thermo-junction of 652° C. The range of the galvanometer scale used was from — 10.00 cm. to + 20.00 cm., or from 552° C. to 841° C., when the cold junction was at 16° C. Thus one millimeter on the scale corresponded very nearly to one degree Centigrade.

When the cylinder had reached a temperature which could be read on the galvanometer, the reading of the galvanometer was noted every minute, and when passing the critical points, readings were made every half minute. The rate of rise or fall varied, but seldom exceeded 20° C. per minute. The current was allowed to pass until the piece had reached a temperature of 800° to 850° C., when the circuit was opened and the cylinder allowed to cool. The cooling was allowed to proceed until a temperature of about 600° C. was reached, when the circuit was again closed and the piece reheated. In general, three heating and three cooling curves were made with each piece.

In every sample there was a point of inflection in the heating curves at a temperature of about 730° C. In piece No. 4 the inflection was slight, the temperature rising continuously but more slowly as this temperature was passed. With the higher carbon steels, however, the temperature at a certain point remained con-

stant, to a fifth of a degree, for a minute or more. Once the temperature appeared to fall about a degree while heating, but the cause of this is uncertain.

In cooling the temperature fell steadily to a temperature of 670° to 710° C., when, except in the case of No. 4, the fall ceased, the temperature rose, remained constant for a period, and then fell steadily. The temperature of recalescence was higher with the higher carbon steels, and the time spent in the process was also

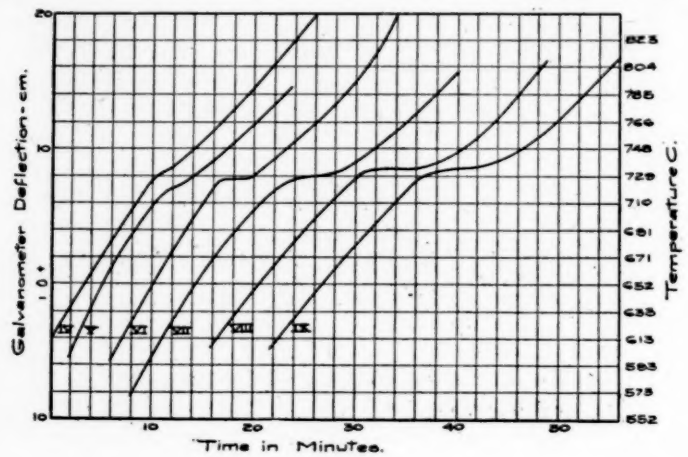


Fig. 4.

greater. In No. 4 there was no recalescence, but the temperature fell decidly less rapidly for a minute or more at about 670° C. The phenomenon observed by Hopkinson[1] and Osmond was also observed, that the temperature of recalescence was lower if the heating had been higher. Some of the cooling curves show a decided bend at the temperature where absorption occurs on heating. There appears to be some change at this point, but it has not been given any special investigation. The heating and cooling curves are given in Figs. 4 and 5.

To see if tempering the steel had any effect, cylinder No. 8 was five times heated to bright redness and plunged into water. It was then rewound and heated as before. No decided change in behavior could be noticed, and the amount of heat absorbed and liber-

[1] Proc. Roy. Soc., Vol. 48, p. 442.

ated in the change of state was, within the limits of error, the same. No. 8 was then heated until its temperature became constant, and

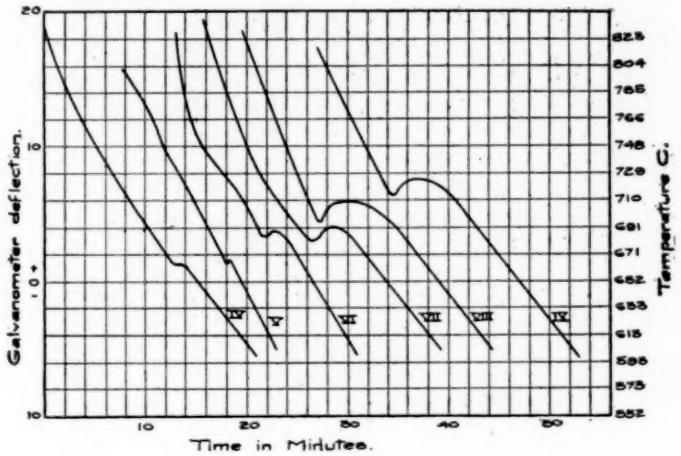


Fig. 5.[1]

just as it began to rise again the current was turned off and the piece began to cool. There was no recalescence, but the temperature

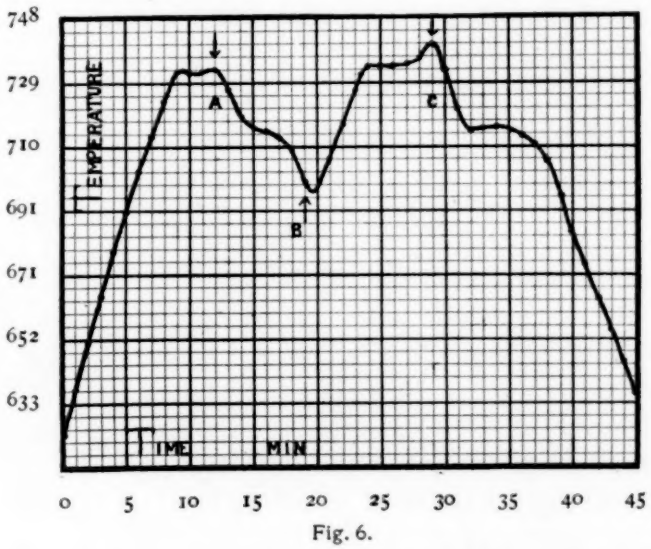


Fig. 6.

[1] By an error in drawing Curve VII has been slightly shifted upward. The form of the curve is correct.

fell rapidly at first, then slowly, then rapidly again. The current was then again turned on and the piece carried to a temperature some 8° higher than before. At this temperature the second rise was well begun. The current was then turned off and the piece cooled rapidly for about 28°, then rose in temperature 1° and finally declined again at nearly the same rate as in the beginning. (Fig. 6.)

The amount of heat absorbed and liberated in the change of state was deduced from the heating and cooling curves by multiplying the mean of the rate before and after the change by the duration of the change and by the specific heat at that temperature as found from the other experiments. In the heating curves, it was hard to decide what tangent to use for the calculation; hence, the values differ widely. In many cases, a tangent was taken parallel to the one previous to the change. The following is a comparative table of the results:

TABLE V.

No. of Piece.	Latent Heat by Calorimeter. gm. cals.	From Heating and Cooling Curves.		Temperatures of—		Temperature Rise in Recalescence.
		Heat Absorbed.	Heat Liberated.	Absorption.	Liberation.	
4	8.6		2.8	729	669	0
5	11.4	3.6	3.6	720	663	2°
6	9.7	8.2	6.5	728	691	4
7	9.7	7.8	8.5	727	661	10
8	16.0	10.9	18.0	733	705	8
9	15.4	10.3	16.0	735	713	7
8 after hardening.			18.2	729, 733	708, 698	9

Comparing the curves for heating and the curves for mean specific heat (Fig. 3), it is seen that the absorption begins in the former very nearly at the temperature at which the latter begin to curve upward. It is seen, also, that the second bend in the specific heat curve is at the temperature where the tangent to the heating curve is parallel to the tangent below the absorption point. These facts, in connection with those of Fig. 6, indicate that the change of state does not all take place at one definite temperature, even if this temperature is maintained for some time. In other words, the substance seems to exist in approximate equilibrium in two states, the proportion of each state being a function of the temperature. Above

the point of absorption, the heating curve gets steeper and steeper, showing that the specific heat in the upper state is lower than that in the lower state. This was indicated by the calorimetric work, but this work was not carried to temperatures sufficiently high to give figures for the specific heat in the upper state.

The lack of agreement in the values of the latent heat, as given by the two methods, is due to the inaccuracy of the method of cooling. In this method the specimen is surrounded by a mass of matter having a heat capacity considerable in comparison with that of the specimen. If this matter could be done away with and the heating and cooling performed with no covering, the results would, no doubt, agree better. To do this, the piece should be wound only with mica and platinum foil and suspended by platinum wires in a receiver exhausted of air. The agreement between the two methods is best where the latent heat is great.

The effect of recalescence in tempering was not investigated, but it is interesting to compare samples 6 and 7. The two have nearly the same percentage of carbon and the same latent heat, but No. 6 recalesces only 4 degrees, while No. 7 goes up 10 degrees. Neither of these are tool steels, but No. 7 can be hardened more than No. 6.

To show the relation between the temperature of a piece of steel and the temperature of its surroundings, when the difference is small, a piece of No. 7 was turned into a ball having a diameter of nearly 2 cm. A hole about 1 mm. in diameter was drilled through the center of this, and the wires of the thermo-element passed through this hole. Care was taken to place the junction of the wires at the center of the ball. The ball was then suspended at the center of the furnace. The standard element above mentioned, was placed close to the side of the furnace. The furnace was then heated and the temperatures of each element noted at regular iuter-vals. In general, the two temperatures rose or fell at the same rate, but at the temperature where absorption occurs, the steel heated less rapidly, and on cooling, the phenomenon of recalescence occurred in the steel while the temperature of the furnace declined uniformly. The two elements were then connected in opposition, and in series with the galvanometer. The readings of the galvanom-

eter were then proportional to differences of temperature between the ball and furnace. The galvanometer deflection was noted every minute while the furnace heated and cooled as before. The readings are shown in Fig. 7, where the ordinates represent galvanometer deflections and the abscissæ represent time. The curve needs no further explanation.

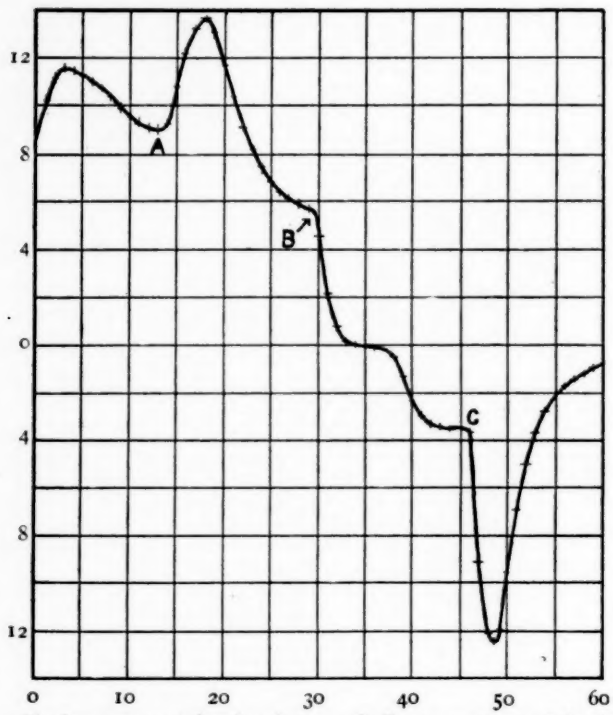


Fig. 7. Abscissæ represent time in minutes. Ordinates represent deflection of galvanometer in cm. and also approximately difference in temperature in Centigrade degree. Positive ordinates represent furnace hotter than iron.

Finally to show the manner in which the furnace heated and cooled, one of the elements was placed in the center of the furnace and connected to the galvanometer as described on page 145. Readings were made every minute, and the results plotted as shown in Fig. 8.

In conclusion, I wish to thank all who by friendly word or generous deed have assisted me during the progress of the work. Special thanks are due to the trustees and faculty of Clark Univer-

sity for providing instruments and means, to Professor A. G. Webster, for continual interest and suggestions, to Mr. A. H. Inman, of

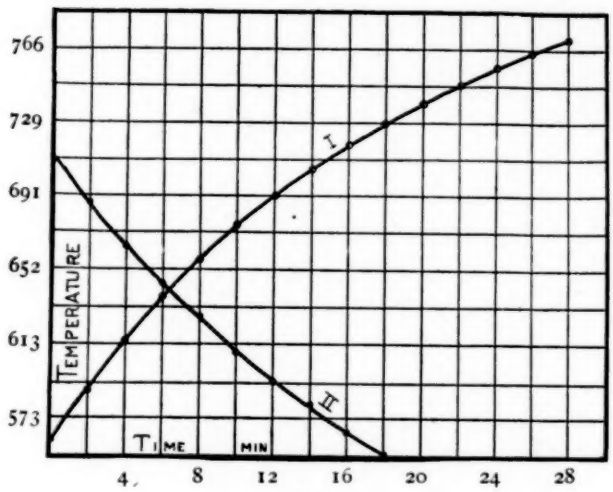


Fig. 8.

the firm of Pratt & Inman for supplying me with samples of steel, and to my fellow student, Mr. E. A. Harrington, for analyzing the samples.

CLARK UNIVERSITY,
WORCESTER, MASSACHUSETTS,
May, 1906.

EXPERIMENTS ON RESONANCE IN WIRELESS TELEGRAPH CIRCUITS. PART V.[1]

XIII. THE ELECTRICAL OSCILLATIONS IN CONNECTED SYSTEMS OF CIRCUITS.

BY GEORGE W. PIERCE.

AS a contribution toward the study of the nature of the oscillation at the sending station used in wireless telegraphy, the following account is given of a series of experiments on the wave-lengths obtained when two condenser circuits inductively or directly connected together are set into oscillation.

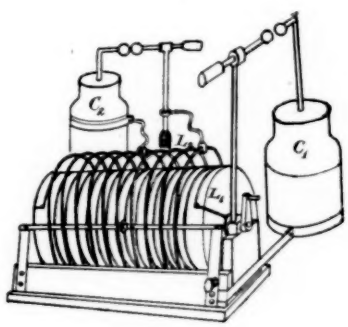
The problem here undertaken is the measurement of the wave-length of each of two condenser circuits with each circuit standing alone, and then a measurement of the resultant wave-lengths when the two circuits are coupled together after the manner of the electromagnetically coupled and the direct coupled sending stations of wireless telegraphy. This problem has already had ample theoretical study, but I am not aware of its having been given extended experimental examination. So the present experiments were undertaken, in part, as a test of the applicability of the theory, and, in part, for the benefit to be derived from an experimental study of specific cases.

The dimensions of the condensers and inductances were chosen to have such values as would bring the wave-lengths within the range ordinarily used in wireless telegraphy (200 meters to 2,000 meters).

Forms of Circuit Studied. — The two types of circuits in which the oscillations were studied are shown in Fig. 49 and Fig. 50. In the *Electromagnetically Connected System* shown in Fig. 49 two condensers C_1 and C_2 are connected to two coils L_1 and L_2 which

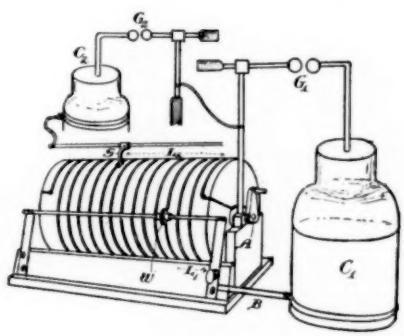
[1] The previous parts were published in the PHYSICAL REVIEW, as follows: Part I., Vol. 19, p. 196, 1904; part II., Vol. 20, p. 220, 1905; part III., Vol. 21, p. 367, 1905; part IV., Vol. 22, p. 159, 1906.

are inductively related but insulated from each other. The number of active turns of wire on each of the coils may be varied. Each of the condenser circuits is provided with a spark-gap, so that either circuit, when connected to a step-up transformer, may be used as the discharge circuit. The other circuit may then be looked upon as a secondary circuit. When the spark-gap of the secondary is opened too wide to permit the passage of a spark, or better when the secondary is removed, the period of oscillation is the period of the primary alone. When, on the other hand, the secondary is left in place and the spark-gap of the secondary is closed, the



INDUCTIVE.

Fig. 49.



DIRECT.

Fig. 50.

oscillations of the discharge circuit are modified by the presence of the closed secondary. It is the purpose of the experiment to measure the wave-lengths produced under these several conditions and to compare the measured values with values computed from theoretical considerations.

In the *Direct Coupled System*, represented in Fig. 50, which was also studied, the transformer of the inductive coupling is replaced by an auto-transformer; that is the two condensers C_1 and C_2 are made to discharge through parts of the same coil. In this case also, both the inductances L_1 and L_2 can be varied independently by the motion of the contacts W and S. Also both the condenser circuits are provided with spark-gaps so that either circuit may be caused to oscillate alone or to constitute the discharge circuit in a connected system with closed secondary.

These two forms of circuits, Figs. 49 and 50, are derived from

the ordinary wireless telegraph circuits by replacing the antenna and ground of the wireless telegraph station by the two coatings of a condenser respectively. The circuits were thus simplified for the purpose of the present experiments because in the simplified form the theoretical formulas ought to apply exactly. In a subsequent investigation it is proposed to study also the less simple cases.

Dimensions of the Inductances.—The coils employed in the apparatus shown in Figs. 49 and 50 had the following dimensions.

Coil.	No. Turns.	Diam. Wire.	Coil Diam.	Pitch.
Outer of Fig. 49	24	.208 cm.	18 cm.	.81 cm.
Inner of Fig. 49	51.5	.208	13	.42
Coil of Fig. 50	51.5	.208	13	.42

The inductances of various numbers of turns of these several coils was measured on a Rayleigh's bridge, and these values are recorded in the tables containing the wave-length measurements given below.

Measurement of Wave-Length. — The measurement of wave-length was made by a resonance method, in which an auxiliary circuit consisting of a variable condenser in series with an inductance was attuned to resonance with the oscillations to be determined. The adjustable auxiliary circuit is called the *wave-meter circuit.*

A wave measuring apparatus based on this principle was described by Drude in 1902 in a paper entitled "Resonanz-methode zur Bestimmung der oscillatorischen Condensatorentladung."[1] In Drude's apparatus the wave-meter circuit consisted of a condenser connected in series with a rectangular loop of wire, whose inductance could be varied by a bridge across the loop. The position of the bridge when the wave-meter circuit was in resonance with the oscillation to be determined gave the wave-length in the oscillation circuit, which could be read off on a scale attached to the wave-meter circuit. Drude ascertained when the wave-meter circuit was in resonance with the oscillation circuit by observing the glow in a Geissler tube attached to one plate of the condenser. The glow is a maximum at resonance.

Several commercial wave-meters based on the principle of reso-

[1] P. Drude, Ann. d. Phys., 9, p. 611, 1902.

nance have been constructed, prominent among which are the wave-meter of Doenitz and the wave-meter of Fleming.

In both of these wave-meters, as in the apparatus of Drude, the essential parts are a condenser and inductance in series, one or both of which may be varied in a practically continuous manner, some means of ascertaining when the wave-meter circuit is in resonance with the oscillations to be determined and a scale from which the wave-length corresponding to the resonant adjustment may be read.

In the Doenitz wave-meter[1] the adjustment of the wave-meter circuit is made chiefly by the variation of the capacity of the condenser, and the resonant condition is ascertained by observing the deflection of a Riess hot-wire air thermometer.

In Fleming's apparatus,[2] both the capacity and the inductance are varied in a practically continuous manner, and the resonant condition is determined by observing the glow in a Geissler tube.

On account of the feebleness of some of the maxima to be investigated in the present experiments it seemed doubtful if either of these wave-measuring instruments is sufficiently sensitive for the measurements required, so that an apparatus was constructed for the purpose.

The Wave-Meter. — One form given to the wave-metric apparatus is shown in Fig. 51. *C* is a variable air condenser of two

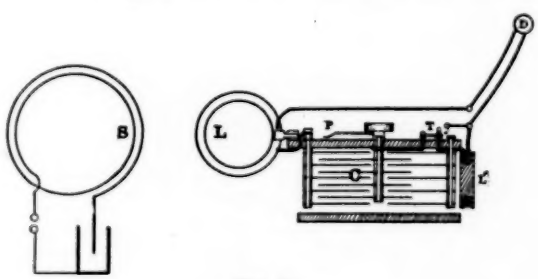


Fig. 51.

sets of semi-circular plates, one of which may be rotated to vary the capacity. The position or the movable plates is given by the reading of a pointer *P* passing over a semi-circular scale. This

[1] Doenitz, E. T. Z., 24, pp. 920-925, 1903.

[2] J. A. Fleming, Phil. Mag., 9, p. 758, 1905. See also Principles of Electric Wave Telegraphy, London, 1906.

scale is divided in circular degrees, and is also calibrated in wave-lengths by the method described below. In series with the variable condenser is a loop of inductance L (8 turns 13.5 cm. in diameter) and the high-frequency dynamometer with variable sensitiveness D described in Parts I. and II. The loop L, is the "receiving" loop to be acted on inductively by the oscillations to be measured, and the dynamometer is for the purpose of determining when the wave-meter circuit is in resonance with the required oscillations.

The wave-meter circuit is also provided with a switch T, by means of which a second inductance L', may be thrown in or out of the circuit. For short waves, between 200 meters and 650 meters, L' may be thrown out, while for waves between 550 meters and 1,800 meters, L' may be thrown in. This gives an easy method of extending the range of the instrument, which must, of course, bear a double calibration giving two wave-length scales, one to be used with L' in, the other with L' out.[1]

The manner of using the apparatus is as follows: If, for example, it was desired to determine the wave-length of the oscillation in the circuit S, Fig. 51, the loop L was placed in such a position that the currents in S acted inductively on the loop L, and gave a deflection of the dynamometer. By observing the deflection of the dynamometer for different adjustments of the variable capacity C, the value of the capacity that put the wave-meter circuit in resonance with the required oscillations could be determined, and from the calibration on the scale of the capacity, the wave-length of the required oscillation could be read off directly in meters.

The apparatus in this form may be extremely sensitive, or its sensitiveness may be decreased to any degree by one of two methods: first, by decreasing the sensitiveness of the dynamometer by drawing the coil of the dynamometer away from the suspended disc, or second, by rotating the receiving loop L about a horizontal axis so that the inductive action upon it is diminished.

Only one precaution is necessary in the use of such a resonance method of determining wave-lengths, namely, care must be taken

[1] Dönitz's wave meter has a triple range, attained by replacing the loop L by loops of higher or lower inductance.

that the inductive coupling between the wave-meter circuit and the discharge circuit be so " loose " that the reaction of the wave-meter circuit on the oscillation circuit is negligible. On account of the high sensitiveness of the present apparatus, there was never any temptation to bring the wave-meter circuit too near to the oscillation circuit.

In some of the experiments described below the dynamometer of the apparatus above described was replaced by other instruments for detecting the resonant condition, but the description of the other instruments is for the present deferred.

Experiment XIX.: Calibration of the Wave-Meter. — The wave-meter was calibrated by an experimental method as follows: A glass-plate condenser in series with a variable inductance and spark-gap was set up in such a position that the spark across the spark-gap could be photographed with the aid of a revolving mirror after the manner of Feddersen's original experiment. The speed of revolution of the mirror was kept constant by the aid of a stroboscope [1] operated by a tuning fork of known period. From the speed of revolution of the mirror and the distance apart of the impressions on the photographic plate the time of one oscillation was determined. This time multiplied by the velocity of light gave the wave-length of the oscillation. The wave-meter circuit was then brought up to within 20 or 30 cm. of the oscillation circuit, adjusted to resonance with it, and the resonant point of the scale of the wave-meter was marked with the wave-length determined by the photograph. In this way a large number of points on the wave-meter scale were determined, and from these points, with the aid of a curve, the wave-length corresponding to any value of the wave-meter scale could be obtained. It was found convenient to write these wave-lengths upon the scale of the wave-meter and to make subsequent readings directly in wave-lengths.

To show the probable accuracy of this method of calibrating the wave-meter some of the data from the measurements of the photographs are given in Table XVI.

In these experiments the speed of the mirror was 41.3 revolu-

[1] G. W. Pierce, On the Cooper-Hewitt Mercury Interrupter, Proc. Am. Acad., XXXIX., No. 18, Feb., 1904.

TABLE XVI.

Photographic Determination of Wave-Length. Capacity Four Boxes of Flat Plates.

No. Turns Inductance.	.5	2.5	4.5	6.5	8.5	10.5	13.5
Inductance in 10^{-5} Henrys.	.50	1.12	2.20	3.43	5.05	6.66	9.38
Distance in cm. between oscillations on plate.	.1096	.164	.227	.301	.340	.404	.483
	.1080	.165	.224	.290	.352	.402	.481
	.1094	.164	.230	.290	.343	.403	.488
	.1103	.160	.238	.292	.343	.404	.484
	.1094		.230	.292	.342		.475
	.1091		.229		.350		
	.1108		.230		.353		
	.1097				.342		
					.346		
Average.	.1095	.163	.230	.293	.344	.403	.482
Wave-Length Meters.	416	630	875	1115	1320	1530	1835
Mean Error per cent.	.5	1.0	1.0	1.0	1.0	.2	.7

tions per second. The distance from the mirror to the sensitive plate was 153 cm., so that the image of the spark travelled across the photographic plate with a speed of $41.3 \times 4\pi \times 153$ cm./sec. = $.792 \times 10^6$ mm./sec. Multiplying the reciprocal of this quantity by the velocity of light in meters/sec., we have

1 mm. between oscillations = 380 meters wave-length.

In determining the distance between oscillations on the photographic plate the length of a whole series of oscillations was measured and divided by the number of oscillations. The table shows that this part of the work was done with a mean error of about 1 per cent.

Besides the values given in the table, the wave-lengths 463 meters and 350 meters were also determined photographically.

Experiment XX.: Extending the Calibration to Shorter Wave-Lengths. — With the particular apparatus at my disposal it was difficult to photograph oscillations giving rise to wave-lengths shorter than 350 meters, so that wave-lengths below this value were obtained by partitioning the condenser. This was done as follows: By the aid of the photographs the wave meter has already been calibrated from 350 meters to 1,835 meters. Using such values of the inductance in the discharge circuit as will bring the

wave-length always within this range the following table (Table XVII.) was obtained by discharging each of four condensers and then the four together through various inductances.

TABLE XVII.

Wave-Lengths with Different Condensers Separate and Together in Discharge Circuit.

Turns of Inductance.	Wave-Length in Meters Given with Condenser.				
	1.	2.	3.	4.	All Together.
4.5	430	415	450	440	875
6.5	555	533	575	570	1115
8.5	675	640	700	685	1320
10.5	790	750	815	800	1530
12.5	895	850	905	870	1780
13.5	950	895	970	935	1835

If now we divide the wave-lengths in columns marked Condenser 1, 2, 3, 4 respectively by the wave-length in the last column of Table XVII., we obtain the ratio of the wave-lengths with the separate condensers in the discharge circuit to the wave-length with the four condensers together in parallel in the discharge circuit. These relative values are compiled in Table XVIII.

TABLE XVIII.

Wave-Lengths with Separate Condensers Relative to Wave-Length with Aggregate.

Turns Inductance.	Condenser.			
	1. λ_1/λ	2. λ_2/λ	3. λ_3/λ	4. λ_4/λ
4.5	.492	.474	.514	.502
6.5	.492	.477	.515	.511
8.5	.502	.477	.521	.510
10.5	.517	.490	.532	.522
12.5	.501	.477	.508	.490
13.5	.517	.488	.528	.508
Mean	.507	.480	.520	.507
Mean error	2 per cent.	1 per cent.	1.4 per cent.	1.4 per cent.

The ratio in each column of Table XVIII. is approximately constant whatever the inductance in the discharge circuit. On the assumption that these ratios are also constant when still smaller inductances are used, as they should be from Thomson's formula, we can extend the calibration to the necessary shorter

waves. For example, the four condensers discharging together through 2.5 turns of inductance gave wave-lengths 630 meters; condenser 1 discharging through the same inductance should give $.507 \times 630 = 320$ meters. Condensers 2, 3 and 4 respectively give wave-lengths 303, 327 and 320 meters, each of which was used to determine a point on the wave-meter scale. In a similar manner the wave-lengths 200, 211, 217, 245, 259, 266, etc., were found and used in the calibration of the scale of the wave-meter.

Experiment XXI.: Resonance Curves Taken in Calibration of Wave-Meter. — In the preceding paragraphs, it was shown how certain standard discharge circuits were made up, and their wave-lengths determined by revolving mirror photographs of the spark. These standard circuits were used in the calibration of the circuit of the wave-meter. The curves of Fig. 52 show how the standard

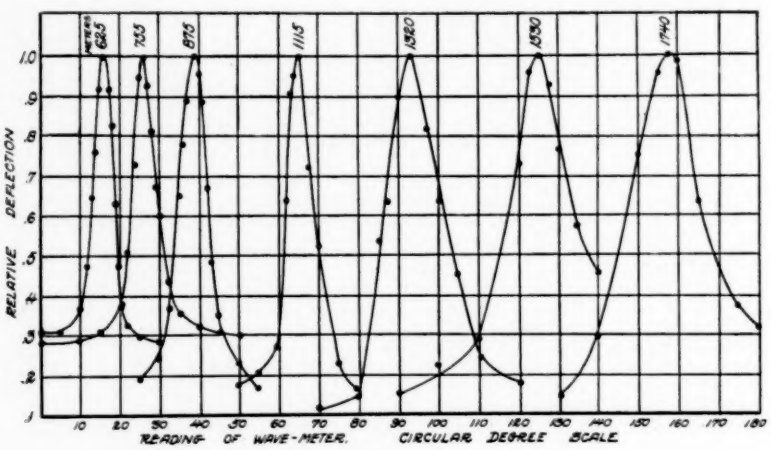


Fig. 52.

wave-lengths were transferred to the wave-meter scale. Here it should be recalled that there are two scales on the wave-meter, one for use when the inductance L' of Fig. 51 was thrown into the wave-meter circuit thus lengthening the period of the wave-meter, the other when the inductance L' was thrown out. The curves of Fig. 52 were taken with the lengthening coil in. For example, with the discharge circuit arranged to give out wave-length 625 meters, the first curve at the left was constructed by plotting the deflections of the dynamometer against the readings of

the circular degree scale attached to the variable condenser of the wave-meter. It is seen that resonance is obtained when the condenser is set at 17 degrees of the circular scale. Since the wave-meter circuit was at a distance sufficiently great from the discharge circuit so that the reaction of the wave-meter on the discharge had no appreciable effect in modifying its period, the period of the wave-meter when set at 17 degrees is the period corresponding to the wave-length 625 meters. In a similar manner from the other curves of the series the number of circular degrees corresponding to the other known wave-lengths was obtained. It is interesting to note that while the resonance curves widen as the wave-length is increased the interval on the scale comprehended within a given number of meters also widens, so that the percentage accuracy of

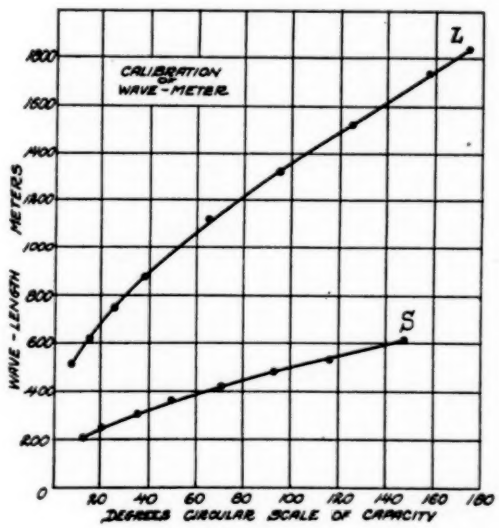


Fig. 53.

the settings of the wave-meter is about the same throughout the scale. An idea of the accuracy of the settings will be had from an examination of the results obtained in the applications that come later in the paper.

If we plot the wave-length at the vertices of Fig. 52 against the reasonant value in circular degrees on the scale of the condenser of the wave-meter, the curve marked L in Fig. 53 is obtained. In like

manner when the switch on the wave-meter is thrown so as to cut out the lengthening coil L', and the discharge circuit is made to produce the known shorter waves, the curve marked S in Fig. 53 is obtained. From these curves respectively two scales were constructed and attached to the wave-meter condenser so that the readings could be made directly in wave-lengths.

Experiment XXII.: Use of the Wave-Meter in the Determination of the Capacity of the Discharge Condenser. — Fleming has pointed out the utility of the waver-meter in the determination of the capacity of a condenser. His method consists of discharging the condenser across a spark-gap through a known inductance and measuring the wave-length by the wave-meter and calculating the capacity from the formula

$$\lambda = v \cdot 2\pi\sqrt{LC}.$$

This method was utilized in the determination of the capacity of the several condensers used in the present experiments. To show the probable accuracy of the method, the following table (Table XIX.) is given showing the results obtained for Leyden jar No. 45 used in the later experiments. The values of the inductances were measured on a Rayleigh's bridge.

TABLE XIX.

Determination of Capacity by the Wave-Meter. Leyden Jar No. 45.

Inductance in Discharge Circuit in Henrys.	Wave-Length in Meters.	Capacity in Farads Computed by Thomson's Formula.
3.10×10^{-5}	690	$.00432\times10^{-6}$
4.90	865	.00432
6.61	1005	.00430
8.35	1130	.00432
10.0	1235	.00430
12.0	1345	.00427
14.05	1450	.00418
16.1	1560	.00423
		Mean $.00428\times10^{-6} \pm 1$ per cent.

The mean error in the measurement of the capacity recorded in Table XIX. is about 1 per cent. This is perhaps better than we ought to expect of the method because in the calculation of the capacity the error in the measurement of the wave-length has been doubled.

Turning our attention now to the main problems of the investigation let us study the oscillations in, first, The Electromagnetically Connected System of Circuits and, second, The Direct Connected System of Circuits.

XIV. Wave-Metrical Study of the Oscillations in the Electromagnetically Connected System of Circuits.

This form of oscillation system is represented in Fig. 49. The theory[1] of this system has been the subject of numerous mathematical researches, so that only so much of the theory will be given here as is necessary in order to enable us to examine the results of the experiments in the light of the theory.

Brief Sketch of the Theory of the Electromagnetically Connected System. — Let the secondary circuit C_2L_2, Fig. 54 be closed, while the primary circuit C_1L_1 is charged and allowed to discharge across a spark-gap.

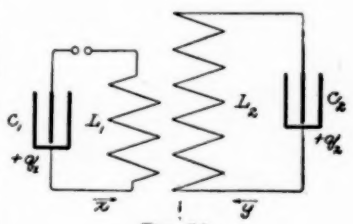


Fig. 54.

Let q_1 be the quantity of positive electricity on the outer plate of the condenser C_1 at any time t during the discharge; let x be the current in the primary circuit at that time; let q_2 be the quantity of positive electricity on the outer coating of C_2, y the current in the secondary circuit: then taking the electromotive force around each of the circuits we have

$$(1) \qquad L_1\frac{dx}{dt} + R_1x + M\frac{dy}{dt} = \frac{q_1}{C_1},$$

$$(2) \qquad L_2\frac{dy}{dt} + R_2y + M\frac{dx}{dt} = \frac{q_2}{C_2},$$

in which L_1 and R_1 are the self-inductance and resistance of the

[1] Lord Rayleigh, Theory of Sound; J. v. Geitler, Sitz. d. k. Akad. d. Wiss. z. Wien, February and October, 1905; B. Galitzine, Petersb. Ber., May and June, 1895; V. Bjerknes, W. A., 55, p. 120, 1895; Oberbeck, Wied. Ann., 55, p. 623, 1895; Domalip and Koláček, W. A., 57, p. 731, 1896; M. Wien, W. A., 61, p. 151, 1897, and D. A., 8 p. 686, 1902; Compare also Webster, Theory of Electricity and Magnetism, p. 499, 1897, and Fleming, The Principles of Electric Wave Telegraphy, p. 209, 1906.

primary, L_2 and R_2 the self-inductance and resistance of the secondary, and M the mutual inductance between the two circuits.

Also

$$x = -\frac{dq_1}{dt}; \quad y = \frac{dq_2}{dt}. \tag{3}$$

Differentiating equations (1) and (2) and substituting from equation (3), we have

$$L_1\frac{d^2x}{dt^2} + R_1\frac{dx}{dt} + M\frac{d^2y}{dt^2} + \frac{x}{C_1} = 0, \tag{4}$$

$$L_2\frac{d^2y}{dt^2} + R_2\frac{dy}{dt} + M\frac{d^2x}{dt^2} + \frac{y}{C_2} = 0. \tag{5}$$

If we eliminate y from these two equations we have

$$(L_2L_1 - M^2)\frac{d^4x}{dt^4} + (R_1L_2 + R_2L_1)\frac{d^3x}{dt^3} + \left(R_1R_2 + \frac{L_1}{C_2} + \frac{L_2}{C_1}\right)\frac{d^2x}{dt^2} + \left(\frac{R_1}{C_2} + \frac{R_2}{C_1}\right)\frac{dx}{dt} + \frac{1}{C_1C_2} = 0. \tag{6}$$

If we eliminate x we get the same equation with x replaced by y.

The equation (6) solved for period of oscillation, on the assumption that the resistance terms have no effect on the period, give the result that the current in the primary oscillates with double periodicity, the current in the secondary also oscillates with double periodicity, and the two periods of the primary are the same as the two periods of the secondary.

The two periods obtained by the solution of equation (6) are

$$T_1' = \sqrt{\frac{T_1^2 + T_2^2 + \sqrt{(T_1^2 - T_2^2)^2 + 4\tau^2T_1^2T_2^2}}{2}}, \tag{7}$$

$$T_2' = \sqrt{\frac{T_1^2 + T_2^2 - \sqrt{(T_1^2 - T_2^2)^2 + 4\tau^2T_1^2T_2^2}}{2}}, \tag{8}$$

in which

T_1 = period of the primary when standing alone,

T_2 = period of the secondary when standing alone, and

$$\tau^2 = \frac{M^2}{L_1L_2}. \tag{9}$$

τ is called the "coefficient of coupling."

If equations (7) and (8) are each multiplied by the velocity of light, we have, remembering that vT = wave length,

$$(10) \qquad \lambda_1' = \sqrt{\frac{\lambda_1^2 + \lambda_2^2 + \surd(\lambda_1^2 - \lambda_2^2)^2 + 4\tau^2\lambda_1^2\lambda_2^2}{2}},$$

$$(11) \qquad \lambda_2' = \sqrt{\frac{\lambda_1^2 + \lambda_2^2 - \surd(\lambda_1^2 - \lambda_2^2)^2 + 4\tau^2\lambda_1^2\lambda_2^2}{2}}.$$

In these equations

λ_1 = the natural wave-length of the primary alone,

λ_2 = the natural wave length of the secondary alone,

τ = the coefficient of coupling, and

λ_1' and λ_2' are the resultant wave lengths in both primary and secondary when the circuits are coupled together.

Measurement of Wave-Lengths Produced by the Electro-magnetically Connected System. — With the aid of the apparatus described above the oscillations in the electromagnetically connected system, Fig. 49, were studied, by measuring the wave-length of the primary circuit and of the secondary circuit when each stands alone and then the wave-length in each circuit when they are coupled together. The results obtained are compared with computations by the aid of the equations (10) and (11). For the computations we need also to know the self inductance of each circuit and their mutual inductance. These quantities were measured on a Rayleigh's bridge.

Experiment XXIII.: E.M.C. System. L_2 = 24 Turns of Outer Coil. λ_2 = 1,060 Meters. — The results obtained in this experiment are plotted in the curves of Fig. 55. The method of taking the observations is as follows: First the condenser C_2(= .00482 m.f.) was connected about 24 turns of the outer coil (Fig. 49) and was provided with a spark-gap. In this position, with the inner coil thrown out of circuit by disconnecting both plates of its condenser, the wave-length λ_2 was found to be 1,060 meters. Next, with the secondary condenser disconnected, the wave-length of the primary (inner) circuit was determined with its condenser C_1(= .00432 m.f.) connected about 50 turns of the inner coil. This wave-length λ_1 was 1,560 meters. Next, with the primary left unaltered, the secondary was closed by attaching its condensers without spark-gap to the 24 turns of the outer coil. This is the case of the closed secondary,

and when the discharge was established in the primary, the wave-lengths were found to be $\lambda_2' = 710$ meters and λ_1' too great for the wave-meter scale. The value $\lambda_2' = 710$ was plotted against $\lambda_1 = 1{,}560$, Fig 55. Now decreasing the primary inductance to 45 turns, the values $\lambda_1 = 1{,}460$, $\lambda_1' = 1{,}650$ and $\lambda_2' = 680$ were obtained, and the last two values were plotted against the value of λ_1, and so forth.

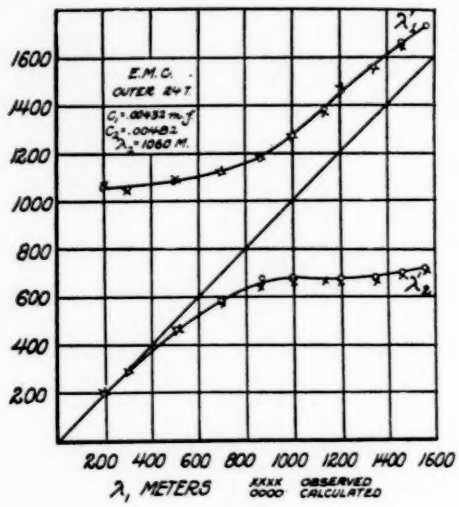


Fig. 55.

The complete record of the observed and calculated values is given in Table XX. In the curves of Fig. 55 the *crosses* are the observed values and the *circles* are the corresponding calculated values. When the observed and calculated values fall together the point is indicated by a *combination* of cross and circle. The 45° line between the two curves may be looked upon as λ_1 plotted against itself, while a horizontal line across the figure at 1,060 meters (not shown) would represent the graph of λ_2. With this in mind it will be seen that the two derived wave-lengths λ_1' and λ_2' are asymptotic toward the origin to λ_2 and λ_1 respectively. The observed and the calculated values are in satisfactory agreement, and their departure one from the other may be due as much to the inaccuracies of the data for the calculated values as to the errors in wave-length measurements.

The formulas for the calculation of λ_1' and λ_2' are the formulas (10) and (11) of page 19, which involve merely the independent periods

TABLE XX.

Electromagnetically Connected System.

Primary capacity .00432 microfarad.
Primary inductance varied.
Secondary capacity .00482.
Secondary inductance 24 turns outer coil, $L_2 = 6.60 \times 10^{-5}$ Henry.
Wave-length of secondary $\lambda_2 = 1060$ meters.

Turns Primary.	L_1 Primary Inductance. Henry.	M Henry.	τ^2
50	15.85×10^{-5}	6.52×10^{-5}	.412
45	13.9	6.14	.421
40	11.8	5.80	.430
35	10.0	5.12	.397
30	8.20	4.45	.360
25	6.50	3.56	.295
20	4.82	2.70	.228
15	3.15	1.95	.183
10	1.72	1.20	.128
5	.69	.47	.048
3	.32	.23	.0277

Turns Primary.	λ_1 Meters.	Calculated.		Observed.	
		λ_1' Meters.	λ_2' Meters.	λ_1' Meters.	λ_2' Meters.
50	1560	1740	727		710
45	1460	1670	712	1650	685
40	1350	1567	686	1570	665
35	1230	1462	680	1480	660
30	1130	1390	660	1370	660
25	1000	1273	685	1280	660
20	870	1185	680	1185	630
15	700	1127	595	1125	565
10	510	1080	467	1090	460
5	300	1060	292	1040	285
3	210	1062	193	1075	210

of the two circuits and their coefficient of coupling. The latter quantity was obtained by the measurement on a Rayleigh's bridge of L_1, L_2 and M for each setting of the oscillation circuit. The values of these inductances and of the values of τ calculated from them is also included in Table XX.

The intensity of the various periods of the circuits under the different conditions of the experiment varies greatly. No attempt was made to record these intensities, since it was found that their

relative values depended on the position of the receiving loop of the wave-meter circuit with respect to its relative distance from the primary and secondary circuits. It is proposed, however, to examine these intensities in a later investigation in which the wave-meter circuit is to be placed at a great distance from the discharge circuit.

Experiment XXIV.: E.M.C. System. $L_2 = 15$ Turns of Outer Coils. $\lambda_2 = 775$ Meters. In a similar manner an experiment was performed with λ_2 constantly equal to 775 meters, obtained by connecting C_2 about 15 turns of the outer coil. The results are plotted in Fig. 56, from which a comparison of the observed and

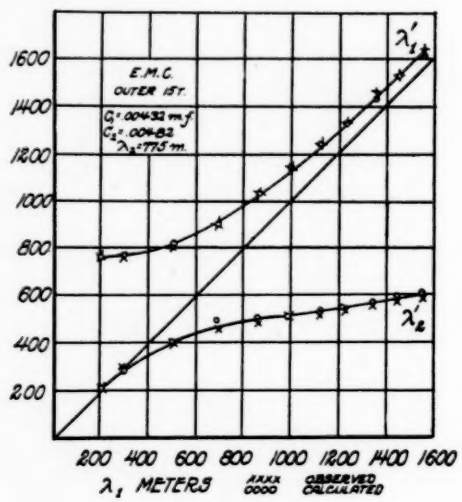


Fig. 56.

calculated values may be had. Here also the agreement is within the limit of error of the method. An abbreviated record of the experiment is given in Table XXI.

Experiment XXV.: E.M.C. System. Special Case $\lambda_2 = \lambda_1$. — A case of especial interest is the case in which the primary and secondary have the same independent periods. This is the case of so-called "resonance" between the two circuits. It is of interest because in this case the wave-length formulas (10) and (11) become greatly simplified, as may be seen by substituting $\lambda_2 = \lambda_1$ in these equations, which under this condition become

$$(\lambda_1')^2 = \lambda_1^2(1 + \tau) \tag{12}$$

$$(\lambda_2')^2 = \lambda_1^2(1 - \tau). \tag{13}$$

In the present experiment the two independent wave-lengths λ_1 and λ_2 were made equal, and the wave-lengths produced by the compound system were then measured and compared with calculations from the formulas (12) and (13). To make the two wave-lengths of the separate circuits the same, the wave-lengths obtained with the

TABLE XXI.

Electromagnetically Connected System.

Primary capacity .00432 microfarad.
Primary inductance varied.
Second capacity .00482.
Secondary inductance 15 turns outer coil, $L_2 = 3.53 \times 10^{-5}$ Henry.
Wave-length of secondary $\lambda_2 = 775$ meters.

Turns Primary.	τ^2	λ_2 Meters.	Calculated.		Observed.	
			λ_1'	λ_2'	λ_1'	λ_2'
50	.295	1560	1610	627	1650	600
45	.317	1460	1530	612	1540	580
40	.345	1350	1440	590	1460	560
35	.377	1230	1337	560	1345	550
30	.396	1130	1255	545	1240	525
25	.390	1000	1152	522	1150	510
20	.354	870	1040	518	1050	490
15	.305	700	918	491	900	465
10	.252	510	830	410	800	410
5	.107	300	780	281	760	300
3	.062	210	774	206	775	210

$\tau^2 = \left(\frac{M^2}{L_1 L_2}\right)$ was obtained by measurements of L_1, L_2 and M.

capacity C_1 discharging through various turns of the inner coil were measured, with the secondary removed, and the curve of wave-length against turns was plotted. In a similar manner the curve of wave-length against turns was obtained for the outer coil with the inner coil disconnected from C_1. From these two wave-length curves, the number of turns of the inner coil that with its condenser acting alone gave the same wave-length as a given number of turns of the outer coil with its condenser acting alone could be selected. The two circuits were now allowed to oscillate together. That is, the primary condenser C_1 was allowed to discharge

through L_1 and the spark gap, while the gap in the secondary circuit was closed, so that oscillations were also set up in the secondary. Two wave-lengths λ_1' and λ_2' were obtained. The results are recorded in Table XXII., and plotted in the curves of Fig. 57.

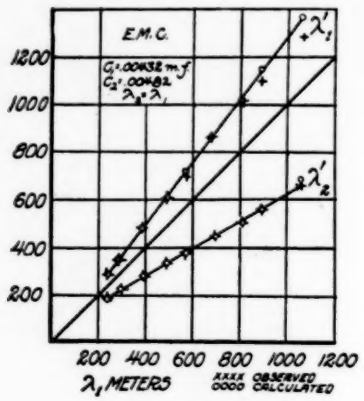


Fig. 57.

TABLE XXII.

Electromagnetically Connected System. Special Case $\lambda_1 = \lambda_2$.

Turns Inner Coil.	Turns Outer.	$\lambda_1 = \lambda_2$ Meters.	Observed.		Calculated.	
			λ_1'	λ_2'	λ_1'	λ_2'
27	24	1060	1290	655	1335	680
19.2	19	900	1095	555	1150	555
18	16	810	1025	503	1032	507
14.9	13	690	860	450	870	440
11.5	10	570	700	380	714	374
9.5	8	487	600	330	609	322
7.3	6	395	480	280	485	278
4.9	4	290	345	224	352	210
3.6	3	252	275	204	294	174

The values of the inductances, used in computations, were

L_1 Henry.	L_2 Henry.	M Henry.	r^2
6.70×10^{-5}	6.95×10^{-5}	4.02×10^{-5}	.348
4.90	4.45	2.90	.386
3.87	4.10	2.30	.374
2.87	3.00	1.72	.343
1.95	2.00	1.14	.323
1.40	1.55	.83	.316
.92	1.10	.50	.253
.45	.64	.257	.227
.21	.40	.14	.230

In this case also the agreement is satisfactory with the exception of the two largest values of λ_1'. No significance is attributed to the departure of these two observations from the theoretical values, as no tendency to a departure of this character was noted in the more general cases described above. It may be that errors were made in reading the wave-meter for these values or, as is more probable, the selection of the turns to make λ_1 and λ_2 equal was erroneous.

The two curves of Fig. 57 are approximately straight lines, which do not pass through the origin. This is due to the particular manner in which τ happens to vary. Constant τ would make the lines pass through the origin.

From Experiments XXIII., XXIV., XXV., with this system of circuits, it is seen that the theoretical formulas give accurately values of the wave-lengths in the electromagnetically connected system, and that the values of the coefficient of coupling required in the equations are accurately enough given by measurements of self and mutual inductance with the comparatively slow frequencies (500 to 1,200 per second) used with an ordinary induction bridge. The theoretical and experimental investigations given below show that the same formulas apply also to the Direct Connected System of circuits.

XV. Wavemetrical Study of the Oscillation in the Direct Connected System of Circuits.

This form of oscillation system is represented in Fig. 50 and in diagram in Fig. 58. The experiments given below show that so far as concerns the wave-length of oscillations exactly the same formulas apply as have been found to be applicable to the electromagnetically connected system. The following sketch of the theory of this system of circuits leads also to this result.

Sketch of the Theory of the Direct Coupled System of Circuits. — In Fig. 58, let the circuit C_1L_1 be looked upon as the discharge circuit, and the circuit C_2L_2 be looked upon as a secondary circuit.

At any time t, let the charge on the outer plate of the con-

denser C_1 be q_1, let the current flowing from this plate at this time be i, and let the current in the coil L_1 be x. Let q_2 be the charge on the outer plate of C_2, and y the current flowing towards this plate: then

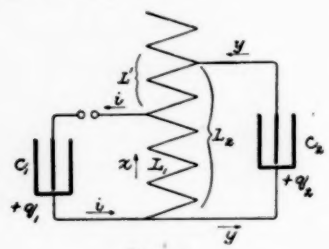


Fig. 58.

$$(14) \qquad i = x + y,$$

$$(15) \qquad q_1 = -\int i\,dt,$$

$$= -\int (x+y)\,dt,$$

$$(16) \qquad q_2 = \int y\,dt.$$

Let L_1 be the self-inductance of the discharge circuit, L_2 the self-inductance of the secondary circuit, and L' the self-inductance in that part of L_2 that is not common to L_1, and let M' be the mutual inductance between L_1 and L'.

Let us suppose that the inductances are localized in the coils, and the capacities localized in the condensers.

If we neglect the resistances of the two circuits, which have but small influence on the period of the circuits, and take the electromotive force around the circuits C_1 L_1 and C_2 L_2 we have

$$(17) \qquad L_1 \frac{dx}{dt} - M' \frac{dy}{dt} - \frac{q_1}{C_1} = 0,$$

$$(18) \qquad L_1 \frac{dx}{dt} + M' \frac{dx}{dt} - L' \frac{dy}{dt} - M' \frac{dy}{dt} - \frac{q_2}{C_2} = 0.$$

Replacing q_1 and q_2 by their values from (15) and (16), we have

$$(19) \qquad L_1 \frac{dx}{dt} - M' \frac{dy}{dt} + \frac{1}{C_1}\int (x+y)\,dt = 0,$$

$$(20) \qquad (M' + L_1)\frac{dx}{dt} - (L' + M')\frac{dy}{dt} - \frac{1}{C_2}\int y\,dt = 0.$$

In order to eliminate x from these two equations let us take $(M' + L_1) \times$ equation (19) and add it to $(-L_1) \times$ equation (20), giving

$$(21) \qquad (L'L_1 - M'^2)\frac{dy}{dt} + \frac{M' + L_1}{C_1}\int x\,dt + \left(\frac{M' + L_1}{C_1} + \frac{L_1}{C_2}\right)\int y\,dt = 0.$$

The second derivative of (21) with respect to t, added to $-1/C_1 \times$ equation (20), gives

$$(22)\ (L'L_1 - M'^2)\frac{d^3y}{dt^3} + \left(\frac{L_1}{C_2} + \frac{L' + M_1 + L_1}{C_1}\right)\frac{dy}{dt} + \frac{1}{C_1C_2}\int y\,dt = 0,$$

which differentiated again gives

$$(23)\ (L'L_1 - M'^2)\frac{d^4y}{dt^4} + \left(\frac{L_1}{C_2} + \frac{L' + 2M' + L_1}{C_1}\right)\frac{d^2y}{dt^2} + \frac{y}{C_1C_2} = 0.$$

This equation may be somewhat simplified by introducing the total inductance of the secondary, L_2, and by expressing the mutual induction in terms of the total mutual inductance between L_1 and L_2 instead of in terms of M' which is the mutual inductance between L_1 and L'. Let the mutual inductance between L_1 and L_2 be M, then by reference to the diagram, Fig. 58, it may be seen that

$$(24)\qquad M = M' + L_1$$

and

$$(25)\qquad L_2 = L_1 + L' + 2M'.$$

From these equations it follows that

$$(26)\qquad L' = L_2 - L_1 - 2M' = L_2 + L_1 - 2M,$$

and

$$(27)\quad L'L_1 - M'^2 = L_1(L_2 + L_1 - 2M) - (M - L_1)^2 = L_1L_2 - M^2.$$

Equations (25) and (27) substituted in equation (23) gives

$$(28)\qquad (L_1L_2 - M^2)\frac{d^4y}{dt^4} + \left(\frac{L_1}{C_2} + \frac{L_2}{C_1}\right)\frac{d^2y}{dt^2} + \frac{y}{C_1C_2} = 0.$$

In like manner, if instead of eliminating x from equations (19) and (20), we eliminate y, we get

$$(29)\qquad (L_1L_2 - M^2)\frac{d^4x}{dt^4} + \left(\frac{L_1}{C_2} + \frac{L_2}{C_1}\right)\frac{d^2x}{dt^2} + \frac{x}{C_1C_2} = 0.$$

This equation is identical with the equation used for the determination of the period of oscillation with the electromagnetically connected system; namely equation (6) with the resistances pu-equal to zero. Whence it follows that the formulas for determining the wave-lengths of the direct coupled system are identical with those for the electro-magnetically coupled system, which are given as equations (10) and (11) on page 19. These formulas are found to apply to the experimental cases that follow.

Experiments with the Direct Coupled System. — Several experiments were made with the direct connected system of circuits. In all these experiments the primary capacity was kept the same, $C_1 = .00432$ m.f. Two different values of the secondary capacity were used, $C_2 = .00178$ m.f. and $C_2 = .00445$ m.f. In each experiment the secondary inductance was set at some fixed value L_2, thus making λ_2 constant, and the primary inductance L_1 was varied from 50 turns to 3 turns, so that the two wave-lengths λ_1' and λ_2' derived when the secondary circuit is closed are functions of the variable primary wave-length λ_1.

Experiment XXVI.: D. C., $C_2 = .00178$, $L_2 = 50.5$ Turns, $\lambda_2 = 1{,}010$ Meters. — The apparatus for the experiments with the direct circuit is shown in Fig. 50. The steps of the experiment

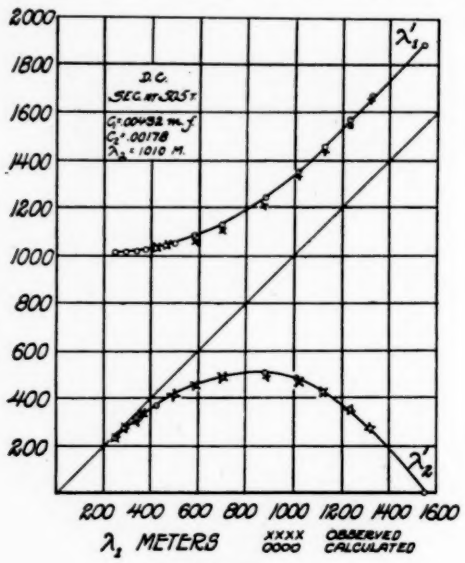


Fig. 59.

are similar to those with the other system of circuits. Table XXIII. contains a record of this experiment. The observed and calculated values of the wave-lengths in the compound oscillating system are plotted in Fig. 59. The formulas of calculation are the formulas (10) and (11), and the only difference between the calculations in the present case and those of the former case is in the method of obtaining M. In the previous case M had to be meas-

TABLE XXIII.

Direct Coupled System.

Primary capacity .00432 microfarad.
Primary inductance varied.
Secondary capacity .00178 microfarad.
Secondary inductance 50.5 turns of coil $= 16.0 \times 10^{-5}$ Henrys.
Wave-length of secondary 1010 meters.

Turns Primary.	L_1 Primary Inductance. Henry.	L'	M	v^2
50	15.85×10^{-5}	$.05 \times 10^{-5}$	15.90×10^{-5}	1.00
45	13.9	.78	14.56	.95
40	11.8	1.85	12.97	.89
35	10.0	3.30	11.35	.805
30	8.20	5.00	9.60	.702
25	6.50	6.66	7.92	.601
20	4.82	8.40	6.21	.498
15	3.15	10.20	4.47	.396
12	2.30	11.30	3.50	.332
8	1.30	12.80	2.22	.237
7	1.05	13.25	1.89	.212
6	.88	13.63	1.62	.113
5	.69	14.00	1.35	.145
4	.50	14.40	1.05	.138
10	1.72	12.00	2.86	.296

Turns Primary.	λ_1 Meters.	Calculated.		Observed.	
		λ_1' Meters.	λ_2' Meters.	λ_1' Meters.	λ_2' Meters.
50		1870			
45	1450	1770	175		
40	1325	1665	272	1650	265
35	1235	1563	355	1540	342
30	1125	1450	427	1435	415
25	1020	1350	477	1325	465
20	880	1240	500	1200	485
15	695	1125	485	1100	485
12	585	1072	450	1050	450
8	425	1030	368	1000	360
7	375	1025	330		335
6	340	1018	317		300
5	290	1018	270		270
4	250	1015	234		235
10	500	1050	405		415

ured on an inductance bridge for every setting of the inductances in the experiments. With the direct coupled system the determina-

tion of M is somewhat simpler. It may be had by calculation from the calibration curve of the inductance coil used, for by equation (26) on page 93 we have

$$M = \frac{L_1 + L_2 - L'}{2}. \tag{30}$$

In this equation L_1 is the inductance of the primary, which may be obtained from the calibration curve of the coil and the number of turns of the primary; L_2 is the inductance of the secondary, had from the number of turns of the secondary; and L', see Fig. 58, is the inductance of that part of L_2 which is not common to L_1. This L' may be obtained as the inductance on the calibration curve belonging to the difference between the number of turns on L_2 and the number on L_1.

The value of M by the use of equation (30) found for each setting of the primary inductance is recorded in the fourth column of Table XXIII. From this value and the corresponding values of L_1 and L_2 a value of $\tau^2 = M^2/L_1L_2$ is obtained, and recorded in the fifth column. λ_1 was measured by the wave-meter for several values of the primary inductance. λ_2 was measured and left constant throughout the experiment. From this value of λ_2 and the several values of λ_1 and τ^2, the values of λ_1' and λ_2' were computed by the formulas (10) and (11). The corresponding values of λ_1' and λ_2' were also observed directly with the wave-meter. The observed values are represented by *crosses* in Fig. 59, and the calculated values by *circles*.

The agreement between the observed and calculated values is within the limit of accuracy of the measurement of the wave-lengths.

Experiment XXVII.: Same as XXVI. Except that $C_2 = .00445$ m.f., and $\lambda_2 = 1{,}575$ Meters. — The capacity about the secondary was replaced by a larger jar giving a longer secondary wave-length of 1,575 meters and observations like those of the preceding experiment were made. The results are plotted in Fig. 60. The curves in this case bear a marked resemblance to those of Fig. 59. The chief differences are that in the latter case with the longer secondary wave-length, λ_1' begins higher up, tangential to 1,575 meters instead of to 1,010 meters as in the preceding case; and the arch of the curve λ_2' is a little higher than in Fig. 59.

In both experiments the curve λ_2' comes down to the horizontal axis in the neighborhood of λ_1 equal 1,575 meters. This point, which was calculated since the wave-meter scale does not extend below 200 meters, is the point of *perfect coupling* indicated by the theory. This value occurred in the experiments when the primary and secondary condensers were both connected about the *same inductance*, 50.5 turns of the coil, so that from the standpoint of the

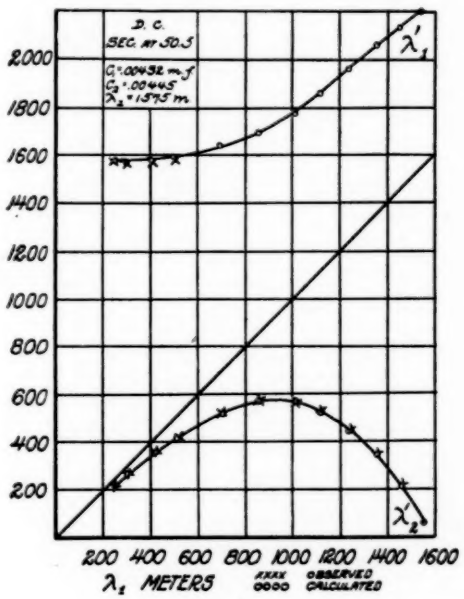


Fig. 60.

experiment the two condensers may be looked upon as discharging in parallel through the same inductance, and producing, therefore, only one wave of wave-length

$$\lambda_1' = 2\pi \cdot v \cdot \sqrt{L_1(C_1 + C_2)} = \sqrt{\lambda_1^2 + \lambda_2^2}.$$

This result is also obtainable from the theoretical equations (10) and (11)

$$\lambda_1' = \sqrt{\frac{\lambda_1^2 + \lambda_2^2 + \sqrt{(\lambda_1^2 - \lambda_2^2)^2 + 4\tau^2\lambda_1^2\lambda_2^2}}{2}},$$

$$\lambda_2' = \sqrt{\frac{\lambda_1^2 + \lambda_2^2 - \sqrt{(\lambda_1^2 - \lambda_2^2)^2 + 4\tau^2\lambda_1^2\lambda_2^2}}{2}}.$$

For when the primary and secondary condensers are connected about the same inductance

$$L_1 = L_2 = M, \text{ therefore}$$

$$\tau^2 = \frac{M^2}{L_1 L_2} = 1.$$

When τ is equal to unity *the coupling is said to be perfect* and the equations given above for the derived wave-lengths become

$$\lambda_1' = \sqrt{\lambda^2_1 + \lambda^2_2}; \text{ and}$$

$$\lambda_2' = 0.$$

That is to say, the oscillation becomes single valued.

The case of perfect coupling was not observed in the experiments with the *electromagnetically coupled system*, because for perfect coupling the primary and secondary coils must have the same number of windings and the two coils must be so close together as to be practically coincident, conditions that could not be realized with the apparatus used in the experiments with the electromagnetic coupling

Other experiments made with the Direct Coupled System of circuits may be epitomized as follows.

	Experiment XXVIII.	Experiment XXIX.	Experiment XXX.
Secondary at	25.5 turns.	25.5 turns.	15.5 turns.
Secondary inductance L_2	6.7×10^{-5} Henry.	6.7×10^{-5} Henry.	3.33×10^{-5} Henry.
Secondary wave-length λ_2	645 meters.	1020 meters.	735 meters.
Secondary capacity C_2	.00178 m.f.	.00445 m.f.	.00445 m.f.
Primary capacity C_1	.00432 m.f.	.00432 m.f.	.00432 m.f.
Primary inductance L_1	varied.	varied.	varied.
Carves.	Fig. 61	Fig. 62	Fig. 63

In all of these experiments the agreement between the observed and calculated values is satisfactory. In the experiments that gave Fig. 61 and Fig. 62 the *point of perfect coupling* occurs at the same value of primary wave-length, about 1,010 meters. This is because the secondaries had the same number of turns in both cases. In Fig. 63 the point of perfect coupling occurs nearer the origin at wave-length $\lambda_1 = 650$ meters. In each of the figures, the curve of λ_2' after passing the zero point again rises toward the right giving again the two periods of oscillation. Lengthy comment on these

curves is unnecessary, since on account of the agreement of the observations with the theory the complete trend of the curves can be obtained from the theoretical equations.

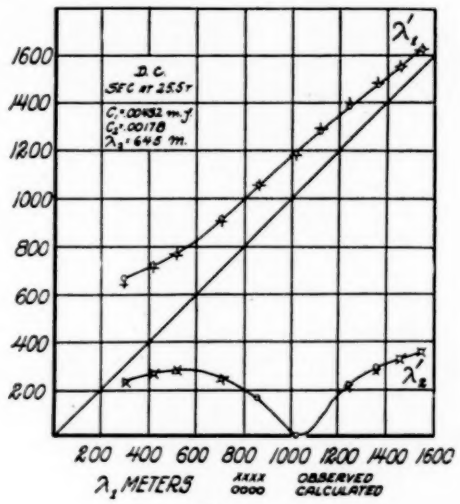


Fig. 61.

Experiment XXXI. : Special Case of the Direct Coupled System, *in which the Primary and Secondary are Adjusted Separately to the same period.* $\lambda_2 = \lambda_1$. — The observed and calculated values in this

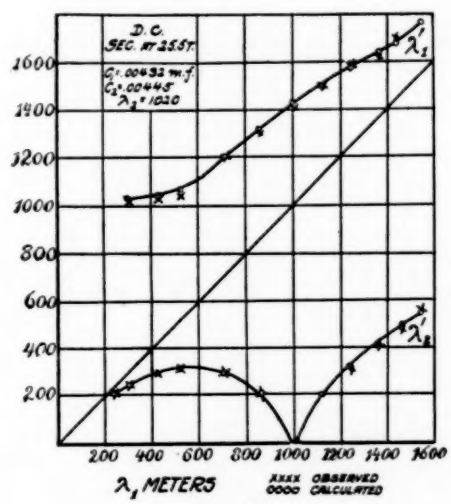


Fig. 62.

case are given in Fig. 64. The figure resembles the corresponding case with the electromagnetic connected system.

In conclusion it should be remarked that the experimental values of the wave-lengths for the entire set of experiments were taken before the calculations were made, and in no case were any of the experimental values redetermined. Each measurement of wave-length was made hastily and with a single setting of the wave-meter No effort was made to obtain high accuracy in the results, as might have been done by averaging several settings of the wave-meter. The results show that the theoretical formulas given above are sufficiently accurate to be used for computing the wave-lengths of the oscillation occurring in connected systems of condenser circuits. Also, on the other hand, the agreement between observed and calculated values show that the wave-meter used is correctly calibrated and is serviceable for the range of wave-lengths between 200 meters and 1,800 meters. It is proposed to utilize the apparatus in direct measurements with wireless telegraph circuits instead of the simpler condenser circuits of the present research.

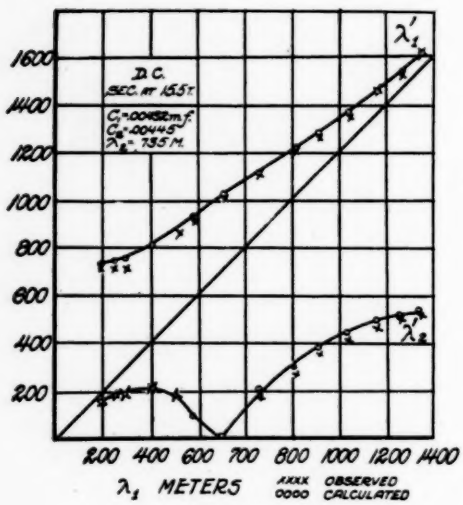


Fig. 63.

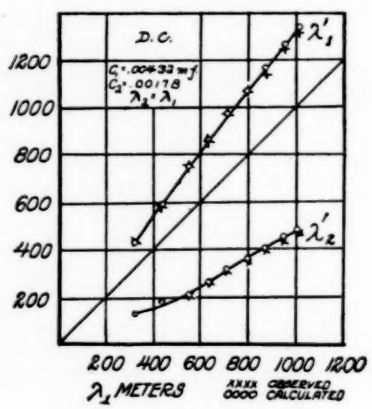


Fig. 64.

JEFFERSON PHYSICAL LABORATORY,
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November 6, 1906.

THE TRANSFORMATION INTO AN ELECTRIC CURRENT OF RADIATION INCIDENT ON A MOVING SURFACE.

By Bergen Davis.

A PLANE polarized beam of light is generally considered to consist of a train of waves in which the electrical component alternates in sign, but remains in the same plane perpendicular to the wave front. As the wave passes through a surface, a point on the surface would be subject to an alternating field of force.

It would be a matter of great importance to rectify the small currents that are set up in a conducting surface by such alternating fields of force. The detection of these currents would not only confirm the electro-magnetic theory of radiation, but would furnish a direct method for distinguishing the electric and magnetic components of the wave.

An account of some experiments by the late Professor A. Bartoli on the direct transformation of radiation into electric currents has recently been published.[1] The writer has lately repeated the experiments of Professor Bartoli, making such changes in the apparatus as our present knowledge of the electronic nature of electric conduction would warrant. Because of the importance of Bartoli's experiment, I think it advisable to give an outline of his reasons for expecting the effect and a translation of that part of his paper which describes his apparatus and the results obtained.

1. Professor Bartoli proceeds from his well-known thermo-dynamic proof of radiation pressure. He performed some experiments seeking to observe the expected radiation pressure, but failing to detect it, concludes that it does not exist. It occurs to him that perhaps his failure is due to the conversion of the incident energy into currents in the conducting surface. The experiment

[1] Su la transformazione in correnti ellectriche delle radiazioni incidenti Sopra una superficie riflettente in movimento. Accademia dei Lincei, 1903, 5th Series, 12.

described below was undertaken for the purpose of detecting these transformed currents.

It is, perhaps, worth while to give a translation of that part of his paper which describes his apparatus and the results obtained.

"Upon a disc *H*, of copper 4 mm. thick and 80 cm. in diameter, a copper band *LPM*, is firmly fixed and perfectly silvered, so that it will reflect at least 92 per cent. of the incident radiation. This band is insulated by means of well-dried pieces of wood, after they have been boiled in linseed oil, to which it is firmly attached by a great number of brass screws.

"The two extremities *L* and *M* of the band are joined by strips

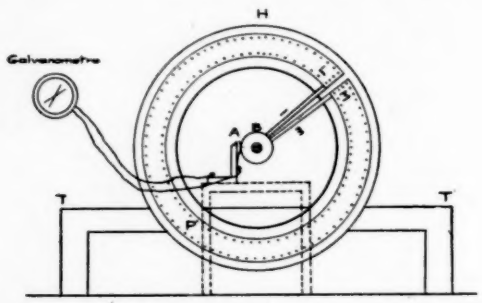

Fig. 1.

of copper, *l* and *m*, to two rings of copper *BB'*, which are 3 mm. thick and insulated by two small ebonite wheels 4 cm. in diameter. The disc *H*, as well as the two small wheels *BB'*, are firmly fixed normally to the steel axis *PS*, which is 3 cm. in diameter but thicker in the middle. Above the two copper rings *BB'*, there are two insulated copper springs *AA'*, which by copper adjusting screws and thick copper wires, communicate with a mirror galvanometer of small resistance and of that type which is completely astatic.[1]

"The axle *PS*, was resting on bronze bearings which were supported by a strong bed frame of a turning lathe of more than 1,000 kilograms and which was firmly fixed to the ground.

"It was possible to impress a very rapid rotation to the disc *H* by means of a belt passing over the iron pulley *P* 4.5 cm. in diameter. The motion was generated by two large wheels at each of which from six to eight men were working. A counting device simi-

[1] A two-needle mirror galvanometer of Magnus, constructed by Sauerwald, Berlin.

lar to Savart's toothed wheel made it possible to count the number of revolutions of the disc in a stated time. After a half-hour's work it was possible to impart to the disc *H* its maximum velocity which was about 100 to 150 revolutions per second. In that case the velocity of a point on the band *LPM* was about 240 to 410 meters per second.

"Before commencing the action, a pencil of solar rays concentrated upon the band *LPM* by means of plane and concave mirrors did not produce any thermo-electric current that made the galvanometer deviate more than two to three scale divisions. During the motion of the disc no thermo-electric current whatsoever (owing to the warming of the rings pressing against the springs, or owing to any other cause) made the galvanometer deviate while the moving disc was in the dark. While the galvanometer needle was motionless and the disc had attained its maximum velocity I let the solar pencil (by pulling up the diaphragm) fall upon the band *LPM* (at about its middle). I could then observe a deviation which at times attained 42 scale divisions. This deviation was always observed. It depended upon the direction of rotation. In fact revolving the disc in the opposite sense at the same speed produced a deviation of about 38 scale divisions in the opposite direction. This deviation was entirely dependent on the velocity of rotation; thus, with a rotatory velocity of about one half that of the maximum a deviation of 20 scale divisions was obtained, *i. e.*, about one half the preceding deviation. The deviation persisted as long as the solar pencil was striking the silvered band; when the rays were cut off by the diaphragm the galvanometer returned to zero; when the diaphragm was pulled up the galvanometer deviated anew.

"These experiments were made in the months of August and September, 1880, in the school of design which is situated on the ground floor of the Technical Institute of Florence. I was assisted in these experiments by Dr. Guido Alessandri, then my assistant, now professor in the Royal Lyceum of Potenza.

"For lack of sufficient motor and local conveniences, I could not continue the experiments which I have only indicated for the sake of recording them. I am satisfied however with the discovery of the fact. I reserve for some other time the repetition of these experi-

ments with improved means in order to determine the laws governing this phenomenon."

It is to be regretted that Professor Bartoli does not give the sensitiveness of his galvanometer. It is impossible to estimate the magnitude of the effect which he observed. It is not probable however that as early as 1880 he possessed an instrument so sensitive as those now at our disposal.

There are some conditions of the experiment just described that make it somewhat doubtful if the effect sought for was really observed. The silver surface was deposited on a copper band of considerable thickness. The light was reflected for the most part and certainly not at all transmitted through the strip. The disturbances produced in the strip would be confined entirely to the surface. The electro-motive force that may have been developed did not extend through the strip, consequently if a current were produced in the surface it would flow around through the back of the strip. The current would be completely closed locally and there could be no effective electro-motive force tending to send a current through the external circuit to the galvanometer.

There are other and more fundamental reasons for the improbability of the real existence of the effect described by Bartoli. Some of these objections will be considered in the second part of this paper.

II. It can be shown however that the alternating currents produced in a moving conductor by incident radiation will not be symmetrical, but during one half period will be of greater or less amplitude than the next half period provided the force acting on an electron is modified by its velocity in an electric field.

Suppose a plane polarized beam of light to pass through a thin conducting film in motion. The plane of the electric component is in the direction of motion. The electro-motive force of the light wave will act through the whole thickness of the film, but its intensity will decrease with the depth owing to absorption. The current produced will be proportional to the electro-motive force at any depth.

Let the sinusoidal form in Fig. 2 represent such a wave propagated downward through a conducting film *CD*. The electric component is in the plane of the paper. Consider that the carriers

of the current are free negative electrons. If the film is at rest as the train of waves passes through, the electrons will be subject to a symmetrically alternating force which will produce a symmetrical alternating current. If the film is in motion the alternating currents will be unsymmetrical or symmetrical according as the force acting on an electron is a function of its velocity in the field or not.

Assume the film to be in motion and the light wave at a given instant to occupy the position represented by case *A*, Fig. 2. The electrons will have a motion relative to the field from left to right due to the motion of the film. This direction is opposite to that which the field of the wave tends to give them at this instant. The second case *B* represents the position of the wave a half period later than case *A*. The motion of the electrons relative to the field is here in the same direction as that which the field tends to produce. The relative motion of the electrons with respect to the field will thus be a variable one. If the force acting on an electron moving parallel to and in an electric field depends on its velocity in the field, the currents in the film will no longer be symmetrical but will have a greater amplitude in one direction. The current will be alternating but unsymmetrical. Since the resistance is constant the effective electro-motive force will also be unsymmetrical.

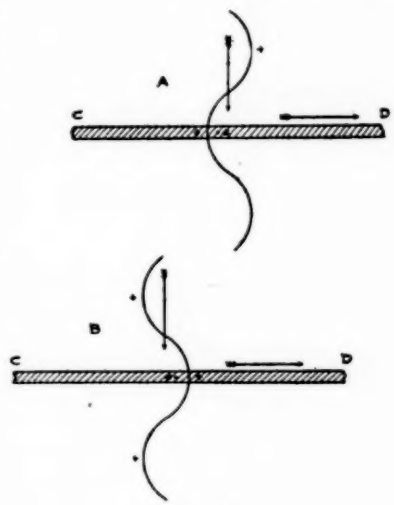

Fig. 2.

I shall make the most simple assumption possible for the dependence of the force on the velocity, namely that it will be modified by a quantity proportional to $(u/v)^2$, where u is the velocity of the film and v is the velocity of light.

Then during the first half period (case A) the effective E.M.F. will be

$$E_1 = X\left[1 + \left(\frac{u}{v}\right)^2\right] \sin pt.$$

The mean value of this is

$$E_1 = \frac{2}{\pi}\left[1 + \left(\frac{u}{v}\right)^2\right] X.$$

During the next half period (case B) the effective E.M.F. will be

$$E_2 = X\left[1 - \left(\frac{u}{v}\right)^2\right] \sin pt,$$

the mean value of which is

$$E_2 = \frac{2}{\pi}\left[1 - \left(\frac{u}{v}\right)^2\right] X.$$

The effective E.M.F. tending to produce a current in one direction in the film is

$$E = (E_1 - E_2) = \tfrac{1}{2}\frac{4}{\pi}\left(\frac{u}{v}\right)^2 X = \frac{2}{\pi}\left(\frac{u}{v}\right)^2 X,$$

where X is the maximum electrical intensity of the sinusoidal light wave. This would be the mean electro-motive force for slow frequencies of vibration. The light wave, however, is of very high frequency, and it is well known that alternating currents of such frequencies cannot be propagated along conductors because of the impedance. Neglecting this impedance the theoretical value of E may be readily found.

Taking the solar constant as 3 gram-calories per minute the energy per cubic-centimeter of sunlight is 2.1×10^6 ergs and the value of X is found to be about 6.2 volts per centimeter.

If the linear velocity of the moving film is 10,000 cm. per second, the ratio $(u/v)^2$ becomes $-\frac{1}{9} \times 10^{-12}$.

Introducing the value of X and $(u/v)^2$ in (3) we have

$$X = 3.6 \times 10^{-11} \text{ volts per centimeter.}$$

If the sunlight is concentrated by a lens to 100 times its natural intensity, the electrical intensity becomes

$$X = 3.6 \times 10^{-10} \text{ volts.}$$

This is the electrical intensity at the upper surface of the film. At the lower surface the electrical intensity will be much less owing to reflection and absorption in the film. Much of the current developed at the upper surface would complete its circuit at the under

surface. The effective electrical intensity would thus be due to the intensity of the radiation that passed through the lower surface.

The electrical intensity (3.6×10^{-10}) is too small to be easily detected on account of the unavoidable disturbances produced by the high velocity of the film.

Also although the electrical intensity $2/\pi(u/v)^2X$ is always in one direction in the film, yet it is not continuous but impulsive, and, as I have pointed out, impulses of the order of frequency of light vibrations can not be propagated along conductors.

In spite of these considerations which indicate an improbability of an effect in view of Bartoli's experiments I have though it worth while to seek to test his observations by a modified method.

Apparatus and Experiment.

The method used was to move a semi-transparent film in a plane perpendicular to a concentrated beam of sunlight falling upon it.

The rotating device for giving a great linear velocity to the film (f) is shown in Fig. 3. An aluminium disc D is mounted on a

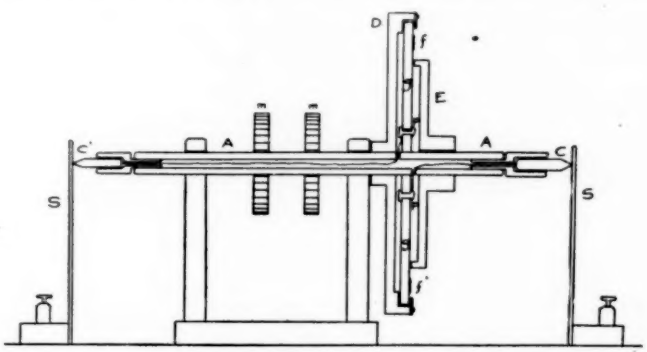

Fig. 3.

hollow steel axle A. The disc is recessed to firmly hold the glass plate g. The disc and plate are rotated by an air motor mm'. Air compresed to about 25 pounds pressure per square inch was directed by suitably constructed nozzles upon the toothed wheels mm'. By this device the glass plate could be given a speed of 200 revolutions per second and the direction of rotation could be quickly reversed.

A thin circular silver film (f) about 8 mm. broad was deposited

by the chemical method upon the glass plate. The strip of silver film formed nearly a closed ring. A radial silver strip extended from each end of the film toward the center and attached to aluminium wires by the small screw bolts *pp'*. These wires (insulated) were carried through the hollow axle to the ends of the shaft where they were attached to aluminium cones *cc'*. These cones were also insulated from the shaft. Aluminium springs *ss'* pressed against these conical points. The points of the cones were made to coincide as nearly as possible with the axis of rotation of the shaft. This reduced the motion between the points of the cones and the contact springs to a minimum.

The greater part of the glass disc was covered by an ebonite cover *E*. This was just large enough to cover nearly all the plate to the inner edge of the film. This cover prevented disturbances arising from the friction of the air against the radial parts of the film and the small bolts *pp'*. Great disturbances were always observed in the galvanometer when the apparatus was used without this cover.

The galvanometer was a D'Arsonval type recently constructed for our laboratory by the Weston Electric Instrument Company. Its resistance is 31.3 ohms. The volt sensitiveness is 81.6×10^{-10} volts per scale division, the scale being one meter from the instrument.

The sunlight was concentrated on the film (*f*) by means of a large lens. The films were of various thicknesses transmitting from .05 to .005 of the incident light. The sunlight could be admitted to or shut off from the film by a movable diaphragm. The greatest rate of rotation attained was about 180 revolutions per second, which gave a linear speed to the film of 6,000 cm. per second.

Very little disturbance was produced in the galvanometer by this rapid rotation. There was occasionally a slow drift, sometimes in one direction sometimes in the other.

A number of films of various thicknesses were experimented with but in no case was there a deflection of the galvanometer that could be traced to the act of admitting the light to the film or to suddenly shutting it off.

Three silver films and one platinum were tried. Their resistances were as follows :

A. Resistances 150 ohms.
B. " 500 ohms.
C. " 200 ohms.
D. This film was formed as follows:

The glass plate was silvered. This silver was coated with a thin film of gelatine. A platinum strip 8 mm. wide was then deposited on this gelatine by the cathode discharge. The platinum film was insulated from the silver by the gelatine. The resistance of the platinum film was 410 ohms.

The order of the effect that might have been expected had there been no impedance already referred to may be approximately calculated. The sunlight was concentrated to about 100 times its natural intensity. The maximum electrical intensity of the electrical component was about 600 volts, of which the resultant components parallel to the direction of motion was 300 volts. The thinnest film transmitted not more than five per cent. of the incident light. The electrical component at the back of the film was thus about 15 volts per centimeter. The velocity of the film was about 6,000 cm. per second. The mean impulsive electrical intensity was

$$x = \frac{2}{\pi}\left(\frac{u}{v}\right)^2 X = 4 \times 10^{-13} \text{ volts per cm.}$$

This was beyond detection by the galvanometer.

With a resistance of 531 ohms in the circuit, one scale division corresponded to an electro-motive force of 1.39×10^{-7} volts.

The effect if it exists was thus about three million times too small to be detected by this instrument.

Conclusion.

Professor Bartoli does not give the sensitiveness of his galvanometer. A direct comparison of the two experiments is consequently impossible. It may however be doubted if at that time (1880) he possessed an instrument of much greater sensitiveness than the one used by the writer. It is true that he attained a greater linear velocity of his conducting strip (40,000 cm. per second). But as I could certainly detect a throw of 4 scale division I should easily have observed an effect that gave him a throw of 42

scale divisions. If the galvanometers were equally sensitive this would allow for a peripheral speed one tenth that obtained by Professor Bartoli.

The fact that with my apparatus, which was on the whole probably as sensitive as that of Professor Bartoli, no effect was observed, and also that, if there was any partial rectification, the electrical intensity must have been impulsive and so destroyed by the impedance, leads at least to the conclusion that the existence of the effect described by Professor Bartoli is not established and must remain in doubt until again observed by other experimenters.

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COLUMBIA UNIVERSITY,
October 27, 1906.

THE ELASTIC MODULUS FOR SMALL LOADS AT THE ELASTIC LIMIT.

By Henry W. Bearce.

IN June, 1899, there was published in the Physical Review a paper by Mr. C. P. Weston, in which was described a series of investigations relating to the application of Young's Modulus when small loads are used. Even with loads of the order of magnitude of 0.5 gram, it was found that the ratio between the stress and the strain holds true.

The present paper describes a series of experiments where the work of Mr. Weston was repeated with the exception that the rods had been previously weighted so as to bring them to their elastic limits.

This was done by applying a load at the center of the rod and increasing this load by small amounts until a permanent "set" could be detected in the rod. A telescope with a micrometer head was used for this purpose and a very slight deviation from the original position of the rod could be detected as the successive loads were removed. The load which caused the first noticeable permanent "set" was taken as the elastic limit and in that part of the experiment in which Young's Modulus was determined, this was used as the initial load to which the 0.5 gram loads were added.

The deflections in the rods were measured by the method of interferences of light waves. The movable mirror of an interferometer was removed and fastened by a pellet of wax to the center of the rod to be tested, the interferometer being placed on its side and illuminated by a sodium flame placed at the right of the apparatus.

The proper height of the mirror was found by raising and lowering the knife edges, and the proper angle by slightly moving the mirror on the pellet of wax and by the adjusting screws of the stationary mirror. By careful adjustment of the distance and angles of the mirrors, straight and clear-cut fringes were found. Then when the 0.5 gram weight was added the fringes were seen to pass

across the field of view. By a count of the number of fringes passing a point of reference the amount of deflection was calculated, assuming as the wave length of sodium light, 0.00006 cm. In order that the fringes could be accurately counted, it was necessary to add the 0.5 gram very slowly and this was done by suspending the

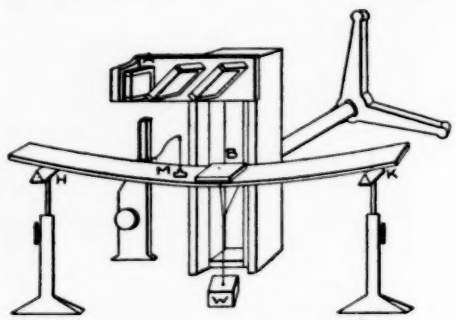

Fig. 1.

weight by a fine silk thread from a horizontal arm moved up and down by a slow motion screw.

The accompanying figure will show the details of the experimental work.

The results of the deflection experiments appear in the table.

TABLE I.

Iron Bar.

No.	Mass in Grams.	Deflection in Wave-lengths.	Deflection in cm.	Deflection for 0.5 Gram.
1	0.560	1.68	0.000101	0.0000902
2	1.025	3.59	0.000215	0.0001048
3	1.585	5.21	0.000312	0.0000985
4	1.995	7.05	0.000423	0.0001083

Elastic load — 6.5 *k*.

Using the common formula for the modulus

$$M = \frac{FL^3}{4bh^3d}$$

F = Wt. in grams × 980.6

L = Length = 60 cm.

b = Breadth = .970 cm.

d = Deflection in cm. for 0.5 gram = 0.000101

$$M = \frac{0.4985 \times 980.6 \times 216000}{4 \times 0.970 \times .125 \times 0.000101} = \frac{105481008}{.000050304} = 20.96 \times 10^{11}.$$

TABLE II.

Steel Bar.

No.	Mass in Grams.	Deflection in Wave-lengths.	Deflection in cm.	Deflection for 0.5 Gram.
1	0.560	1.34	0.0000804	0.0000718
2	1.025	2.44	0.0001464	0.0000714
3	1.585	3.67	0.0002202	0.0000695
4	1.995	5.01	0.0003006	0.0000753

Elastic load — 24.25 *k*.

$L = 60$ cm. $\quad h^3 = .154854$

$b = 1.008$ cm. $\quad d = 0.000720$

$h = .537$ cm. $\quad M = 21.04 \times 10^{11}$.

TABLE III.

Copper Bar.

No.	Mass in Grams.	Deflection in Wave-lengths.	Deflection in cm.	Deflection for 0.5 Gram.
1	0.560	1.35	0.0000810	0.0000723
2	1.025	2.74	0.0001644	0.0000802
3	1.585	4.54	0.0002724	0.0000859
4	1.995	5.92	0.0003552	0.0000891

Elastic load — 13.25 *k*.

$L = 61$ cm. $\quad h = 0.639$ cm.

$b = 1.087$ cm. $\quad d = 0.0000819$

$M = 12.26 \times 10^{11}$.

TABLE IV.

Brass Bar.

No.	Mass in Grams.	Deflection in Wave-lengths.	Deflections in cm.	Deflection for 0.5 Gram.
1	0.56	1.35	0.0000810	0.0000723
2	1.025	2.68	0.0001608	0.0000784
3	1.585	4.02	0.0002412	0.0000768
4	1.995	5.51	0.0003306	0.0000829

El stic load — 19.5 *k*.

$L = 60$ cm $\quad h = 0.638$ cm.

$b = 1.094$ m. $\quad d = .0000776$

$M = 11.59 \times 10^{11}$.

A comparison of the above results with those obtained by Prof. Weston is shown in the next table.

TABLE V.

Comparison of Results under Ordinary Conditions with those at Elastic Limit.

Bar.	At Elastic Limit (interferometer).	Ordinary Conditions (interferometer).	Ordinary Conditions (large loads used).
Steel.	$21.04 \times 10''$	$19.19 \times 10''$	19.56×10^{11}
Copper.	12.26×10	11.87×10	12.00×10
Brass.	11.59×10	10.64×10	10.62×10

It will be seen that the results for the modulus when the bars are at the elastic limit are slightly in excess of those under ordinary conditions.

The accompanying curves show the relation between loads differing by 0.5 gram, and the resulting deflections measured in wave-lengths of sodium light. In the plot each vertical space equals 0.5 gram, and each horizontal space one wave-length.

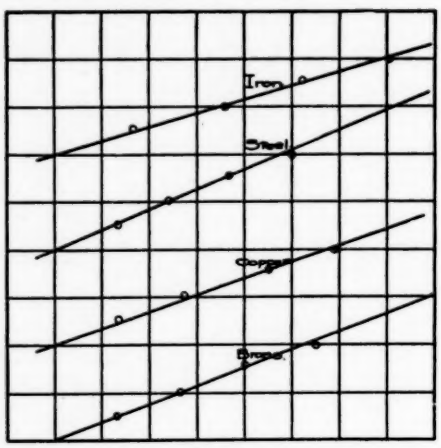

Fig. 2.

In conclusion it may be said:

1. These experiments seem to show a slight increase in the elastic modulus as bars reach the elastic limit.

2. Within the limits of errors of observation Hooke's law holds at the elastic limit as well as under normal conditions.

This work was suggested by Prof. James S. Stevens and carried on under his direction.

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INDEX OF REFRACTION AND DISPERSION WITH THE INTERFEROMETER.

By C. A. Proctor.

TO determine the index of refraction of a dispersion medium by means of an interferometer it is necessary to introduce into one arm of the instrument a known thickness of the substance to be tested, and to count the number of fringes which cross the field while this change in the light path is actually occurring, or else to employ two or more light sources of different wave-length. For if we merely determine, by moving one of the mirrors, the shift of the white light fringes due to the substance, the results obtained will be incorrect owing to the shift due to dispersion.[1]

The usual laboratory method of determining the true value of this fringe shift may be briefly outlined as follows:[2] A plate of uniform thickness of the substance, *e. g.*, glass, whose index is to be determined, is cut in two, and one half is mounted in each arm of the interferometer so as to cover half the field. For this purpose the instrument is provided with two extra frames capable of rotation about vertical axes, one of them being equipped with a slow-motion attachment for rotating. The two pieces of glass are arranged to cover the same portion of the field of view and are set normal to the light paths, and the interferometer is adjusted for white light fringes. When these adjustments are made the fringes in the portion of the field covered by the glass will be continuous with those in the remainder of the field. One glass is now turned through a convenient angle. The other glass is then turned by means of the slow-motion attachment, the fringes of sodium light being counted as they cross the field until the white-light fringes again occupy their former position. The angle through which either glass turns is measured with the aid of a telescope and scale. If this angle is i, the fringe

[1] C. R. Mann, Manual of Adv. Optics, p. 37.
[2] Loc. cit., p. 61.

count $2N$, the thickness of the glass t, the index of the glass μ, and the wave-length of sodium light λ, it may readily be shown that

$$\mu = \frac{(t - N\lambda)(1 - \cos i) + \dfrac{N^2\lambda^2}{2t}}{t(1 - \cos i) - N\lambda}. \qquad (1)$$

The term $N^2\lambda^2/2t$ is negligible.

If we now restore one glass to a position normal to the light path and move one of the mirrors until the white light fringes in the part of the field covered by the glasses once more occupy their former position, the number of sodium fringes crossing the field will be greater than $2N$, the difference, which we will call $2N'$, depending upon the dispersion. If we assume the relation

$$\mu = A + \frac{B}{\lambda^2},$$

we have

$$N' = \frac{2Bt'}{\lambda^3}. \qquad (2)$$

In this expression t' is the thickness of glass introduced into the path by the rotation. In this way we can obtain both the index and the dispersion of our substance, using light of a single wave-length.

The objection to this method is that it demands a sample of glass for test in the form of two plane parallel plates of the same thickness, and a specially equipped interferometer. This makes it in many cases unavailable, either as a practical method of determining indices, or as a laboratory exercise.

It occurred to the writer that the above procedure might be considerably simplified, and that resort to the laboratory scrap pile should furnish materials to satisfactorily take the place of the slow motion attachment on the interferometer for rotating the glass. Accordingly a very rough micrometer screw with its nut was taken from a disused coefficient of expansion apparatus. The graduated disc was removed and in its place was put an arm of brass about 10 cm. long and 0.5 cm. thick. The nut was then secured in a clamp mounted on a heavy laboratory support in such a way that, as nearly as could be judged by eye, the screw was vertical. A

spherometer from which the legs and disk had been removed was held by another clamp on the rod of the same support in such a way that the screw was in a horizontal position with its point bearing against the arm attached to the vertical screw. This contact was maintained with the aid of a spiral spring. The glass to be tested was waxed to the lower end of the vertical screw, and a galvanometer mirror to the upper end. This device was placed so that the glass projected into one path of an interferometer far enough to cover the upper half of the field. It furnished a slow motion for turning the glass which gave complete satisfaction.

The method of experiment employed was as follows. From a fragment of plate glass two pieces were cut approximately 3×3 cm. each, one of which was mounted on the slow-motion apparatus as described above. As is generally the case the surfaces of this glass were not parallel, so that when it was introduced into one arm of the interferometer the fringes disappeared from the field owing to the deviation of the light passing through the glass. This effect could of course be corrected by the readjustment of one of the mirrors but such a procedure would entail the loss of the fringes in the lower part of the field, and even if the surfaces were nearly enough parallel so that the fringes were visible in both portions of the field, they would be at a very low visibility in one if white light fringes were present in the other. As it is convenient, though by no means essential, to have the fringes in the lower part of the field for purposes of reference, the second piece of glass was mounted in the other arm so that it covered the same part of the the field as did the first. This glass was then turned in its own plane until the fringes were of the same width in both portions of the field. This adjustment is possible only when the lack of parallelism of the surfaces of the two squares of glass is nearly the same, a condition which will generally be fulfilled if they are adjoining portions of the same plate. The glass attached to the slow-motion device was then set normal to the light, *i. e.*, in the position of minimum retardation, and the central fringe of the white-light system brought to a fiducial line in the field. The glass was then turned and the sodium fringes counted until any desired number had crossed the field. Following the notation used above we will

call this number $2N$. The angle i through which the glass turned was measured with a telescope and scale. The movable mirror was then shifted and the fringes counted until the central fringe in white light was again on the fiducial line. This count was larger than $2N$ by a number, $2N'$ given by equation (2). That such must be the case appears from the following considerations: If we call the central white light fringe at the start that of order zero, the fringe on the fiducial line after the glass has been turned will be that of order $2N$. If now we move the mirror till $2N$ fringes have crossed the field in the opposite direction the fringe of order zero will have returned to its former position. It will, however, be no longer the center of the white light system because of the shift of that system due to dispersion. As this shift is always in such a direction as to increase the apparent retardation we shall accordingly have to move the mirror further until $2N'$ more fringes have crossed the field before the new central fringe, which is that of order $-2N'$, coincides with the fiducial line.

From N, N', i, and the thickness of the glass t, we can, with the help of equations (1) and (2), compute the index and the dispersion constants of the glass.

Two of the sources of possible error in the above method are worthy of special consideration. These are the lack of parallelism of the surfaces of the glass, and the determination of the angle through which the glass is rotated.

If the faces of the glass are not parallel an error may be introduced owing to the fact that as it is rotated the thickness introduced into the path will not be given by the simple relation on which equation (1) is based, and will be different in different parts of the field. Such a condition will be indicated by a continual change in the width of the fringes as the glass is rotated. This difficulty may be entirely eliminated by so mounting the glass that the line of intersection of its two faces shall be perpendicular to the axis of rotation. In the case of glass No. 2 measurements on which are given below (a fragment of a microscope slide) the general irregularity of the thickness was such that the fringes were very much distorted. However, as the pattern and its position in the field did not alter appreciably during a shift of 150 fringes it may be assumed

that no error in the fringe count is introduced. Of course the thickness must be measured at the point where the fringe shift is observed.

The angle to be measured is the rotation of the glass about an axis in its own plane. If the actual axis is not vertical (we assume the light path to be horizontal) the angle determined with the telescope and scale may not be the one desired. This possibility may be guarded against by setting the scale parallel to the glass at the start, and tilting it until its image in the telescope shows no vertical displacement as the glass is turned. The determination of the scale distance is however a matter of some difficulty, and it is in this measurement that the greatest source of error seems to lie. In the apparatus used by the writer for example the absolute error may well have amounted to a half millimeter or even somewhat more. This might be bettered by having the axis of rotation of the glass more definitely marked.

To illustrate the foregoing some determinations of the index and dispersion constants of two samples of glass were made. No 1 was a piece of plate glass and No. 2 a fragment of a microscope slide. In each case the constants A and B were determined from observations on sodium fringes. As a check the index for the mercury and for the hydrogen line was computed from A and B, and also directly determined using vacuum tubes as the light source. The thickness of the glass was measured with micrometer calipers. In the case of the readings on No. 1 with different scale distances the glass was re-set each time the scale was moved. The results are given in the appended Table.

The method above described may be applied with satisfactory results to samples of glass varying in thickness from 1.5 mm. up. By using polarized light and properly mounting the plate under test it may also be adapted to the test of crystals of a like range of thickness. Another method, however, suggests itself which might be more conveniently used with crystals, and which is adapted to use with much thinner samples of the material. This may be outlined as follows:

The interferometer is adjusted for white-light fringes, and the lamina whose index is to be determined is introduced into one arm

Glass No. 1. *Thickness* = 8.741 *mm.* $A = 1.5102$, $B = 50.5 \times 10^{-10}$.

Source.	First Count $2N$.	Second Count $2(N+N')$.	Scale Distance Cm.	Index Computed From A and B.	Index Determined From N.
Na. $\lambda = 5896$	300.0		55.39		1.5254
"	400.0		55.39		1.5247
"	500.0		55.39		1.5243
"	150.0		75.42		1.5254
"	250.0		75.42		1.5258
"	350.0		75.42		1.5256
"	200.0		68.79		1.5240
"	350.0		68.79		1.5249
"	400.0		69.95		1.5244
"	399.9	414.1	59.90		1.5247
Hg. $\lambda = 5461$	400.6		59.90	1.5271	1.5270
H. $\lambda = 6563$	400.7		59.90	1.5219	1.5221
Glass No. 2. *Thickness* = 1.702 *mm.* $A = 1.5033$, $B = 51.0 \times 10^{-10}$.					
Na. $\lambda = 5896$	110.1	114.1	60.03		1.5180
Hg. $\lambda = 5461$	118.0		60.03	1.5205	1.5212
H. $\lambda = 6563$	100.0		60.03	1.5152	1.5156

so that it covers a portion of the field and is normal to the light. One mirror is then moved till the white-light fringes return to their former position in the part of the field covered by the lamina, the fringes of sodium light which meantime cross the field being counted.

If $2N_1$ = number of fringes counted,
μ_1 = index of substance for sodium light,
λ_1 = wave-length of sodium light,
B = second constant of Cauchy's formula,
t = thickness of lamina,
we have the relation

$$(\mu_1 - 1)t = N_1\lambda_1 - \frac{2Bt}{\lambda_1^2}.$$

If we repeat this using light of wave-length λ_2 (*e. g.*, green mercury light) we shall have

$$(\mu_2 - 1)t = N_2\lambda_2 - \frac{2Bt}{\lambda_2^2}.$$

If now we attach the lamina to one of the mirror frames, we can obtain white-light fringes by reflection from either of its surfaces.

If we count the number of sodium fringes $2N_3$ which cross the field as we pass from one of these white-light systems to the other we shall have

$$\mu_1 t = N_3\lambda_1 - \frac{2Bt}{\lambda_1^2}.$$

N_3 must be corrected for the half wave-length change of phase occurring on reflection at the front surface.

From these three equations and Cauchy's formula the dispersion constants may be readily obtained.

UNIVERSITY OF MISSOURI,
September 20, 1906.

THE LUMINOUS EQUIVALENT OF RADIATION.

By P. G. Nutting.

THE visual response to radiation depends upon the intensity and quality of the radiation and the time during which it is operative, as well as upon various subjective conditions. Some function of energy and wave-length is therefore necessary in order to translate radiation into light or visual brightness, that is, to make possible a purely physical definition of light. The construction of this and subsidiary functions is the subject of this discussion.

The problem is essentially one of determining the properties of the eye as a physical instrument, as a species of radiometer. It is necessary to relate the indications of this instrument to the quality and intensity of the incident radiation, in order to translate exciting stimulus (radiation) into scale reading (light).

The same amounts of radiation in watts per unit wave-length in different parts of the spectrum, affect the eye in different degrees. Hence, other things being equal, the sensibility of the eye is a function of the wave-length, say $V(\lambda)$, call this *visibility* of the radiation. For any arbitrary spectral distribution of radiation $E(\lambda)$ then, the product EV will give a third function of wave-length, say $L(\lambda)$, which we may call the *luminosity* or optical intensity of the radiation. The determination of the visibility function $V(\lambda)$ is the first step toward the solution of the general problem.

Again, the sensibility of the eye varies with the intensity of the incident light. In range, the eye is comparable with an ammeter capable of registering millions of amperes and milliamperes as well. With increase of intensity the sensibility decreases according to some function $S(L)$ of the luminosity. This sensibility function $S(L)$ may be derived from data on the photometric constant and includes Fechner's law as a special case.

Now the sensibility of any instrument at any particular value of the exciting stimulus is the derivative of scale reading with respect to exciting stimulus. Hence, once $S(L)$ is determined, the *visual*

brightness function $B(L)$ is determined by the general integral of SdL. This brightness function $B(L)$ or $B(E, \lambda)$ is the one ultimately desired.

One might proceed further with the sensibility of the eye for differences in wave-length, thus dissecting $V(\lambda)$ into three or more components $V_1(\lambda)$, $V_2(\lambda)$, $V_3(\lambda)$, $\cdots$, but this has little physical significance except in determining the whiteness of light.

The surface integral of the spectral integral of $E(\lambda)$ is evidently the total energy emitted by a source. Similarly the same integrals of $L(\lambda)$ is the total light emitted. This light integral, tempered or not, according to whiteness, is what possesses commercial value in illumination. The ratio of the light integral to the energy integral is the *luminous efficiency* of a source.

The Visibility Function.

The visibility function gives the relative sensibility of the eye to radiation of different wave-lengths. It is by definition ($L \equiv EV$) the ratio of visual intensity L to energy intensity E. It may be determined (*a*) by taking the reciprocal of the threshold value E_0, or (*b*) from the luminosity, determined visually, of a source whose energy curve is known. The identity of visibility with reciprocal threshold value will appear later in the discussion of sensibility as a function of intensity, and is abundantly confirmed by experiment.

Langley[1] measured the least energy by which it was necessary to illuminate a logarithm table to make it legible. A König[2] determined threshold value and luminosity at both high and low intensity. Pflüger[3] measured threshold value with a thermopile. In all, about fifty sets of experimental data on visibility are available. All show a pronounced maximum in the green, a rapid falling off toward blue and red to a very small but finite value in the extreme red and violet.

The position of the maximum of visibility at low intensity varies somewhat with the individual, but the mean of the forty used by the writer[4] is at 510 $\mu\mu$. The maximum lies between 500 and 520

[1] S. P. Langley, Energy and Vision, Am. Journ. Sci., 36, 359–380, 1888.

[2] A. König and C. Dieterici, Z. Psy. Phys. d. Sinnesorgane, 4, 241–348, 1893.

[3] A. Pflüger, Ann. d. Physik, 9, 185–208, 1902.

[4] A more extended treatment of this subject with full quotations of all data used is to appear in the Bulletin of the Bureau of Standards.

for nine out of every ten individuals. Eleven curves from the best data of Langley and König reduced to the same maximum ordinate are reproduced in Fig. 1. All but two of the remaining curves would have fallen within this same galaxy but could not be reproduced without crowding the figure.

The physical problem does not require a special function that will closely represent each individual curve. What is required is a single general function of the simplest possible form and with fixed mean parameters that will represent the properties of the average

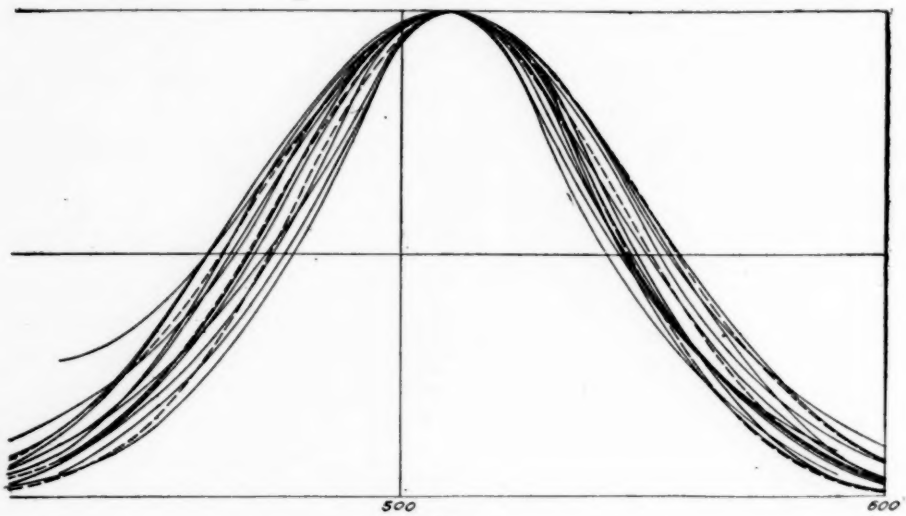

Fig. 1.

human eye as closely as any other function that might be constructed. Special work will always require a special investigation of the properties of the observers eye at the time he is working. General problems of illumination, however, require the specification of the properties of a mean average standard eye.

From the observed data it appears that the mean visibility curve $V(\lambda)$ is very nearly if not quite symmetrical and similar to the probability curve in form. The dotted curves in the figure are the plotted values of the probability function.

$$V = V_1 e^{-a(\lambda-\lambda_m)^2}$$

with $\lambda_m = 5.10$ s.-m. ($\equiv 510\,\mu\mu$) and $a = \pi$, 4 and 5. The para-

meter four represents the mean value very closely. The parameter π would have the advantage of giving the curve unit area per unit maximum ordinate but is obviously too small.

The sole condition thus far imposed upon the parameters V_1, a and λ_m is that they are not functions of wave-length. V_1 depends upon the unit of luminosity chosen; a decreases and λ_m increases somewhat with increase of intensity. The values given, $a = 4$ and $\lambda_m = 5.1$ s.-m., refer to very low intensities approaching the threshold value. The mean visibility curve for low intensities is practically the same for all individuals, being sensibly independent in both form and position of color blindness, partial or complete, and of the absolute sensibility of the individual eye.

The visibility function at high intensities however, varies considerably in cases of partial color blindness. For normal eyes, the only data is that of Kŏnig, on his own and one other eye. The curves are broader than those at low intensity, and displaced toward the red. The visibility function given fits them fairly well for $a = 2$ and $\lambda_m = 5.6$. With such scant data however, these values cannot be regarded as established. The shift of the maximum with intensity may easily be demonstrated by means of a small grating spectograph and sunlight. As the slit is opened the maximum of brightness is seen to move over from deep green toward the yellow.

Another method of obtaining the variation of a and λ_m with intensity suggests itself. Suppose the threshold value of the luminosity L_0 is determined, as was done by Kŏnig and Brodhun,[1] in terms of a rather large light unit, say one meter-candle. If now the visibility function $V(\lambda)$ for the higher intensity were identified with that $V_0(\lambda)$ at low intensity, $L_0(\lambda)$ would be constant. In any case $L_0 = V : V_0$.

In Fig. 2 are shown Kŏnig's values of L_0, the visibility curve V_0 for $a = 4$ and $\lambda_m = 5.1$ together with the curve L given by the product $V_0 L_0$ at each wave length. The curve V' is that obtained by Kŏnig from luminosity and energy at high intensity, but is subject to a large and uncertain correction for the spectral energy variations. It has the same maximum but is much broader than the calculated visibility curve.

[1] Kŏnig and Brodhun, Sitz. Ak. Berlin, July 26, 1888.

Calculating L_0 as the ratio $V : V_0$

$$L_0 = e^{-2(\lambda-5.6)^2} : e^{-4(\lambda-5.1)^2} = e^{2(\lambda-4.6)^2},$$

a curve of the same general form as that obtained directly from König's data. The values $a = 2$ and $\lambda_m = 5.6$ for the parameters of the visibility function at high intensities cannot be regarded as established on the scant data at present available, much less can

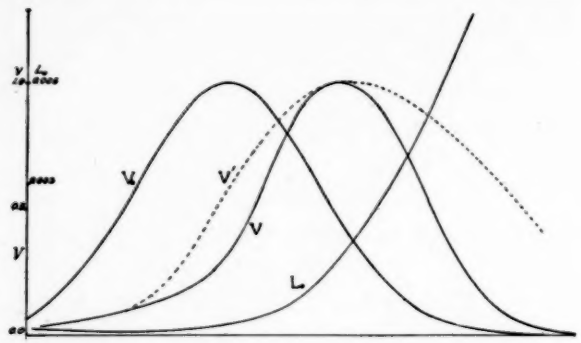

Fig. 2.

the general functions $a(E)$ and $\lambda_m(E)$ be determined as would be required for a complete solution of the general problem.

VISUAL BRIGHTNESS.

The eye will always underestimate actual differences in intensity, other things being equal, on account of the decrease in sensibility with increase in intensity. But since the magnitude of a sensation can be estimated only with the roughest approximation, indirect methods must be resorted to in order to determine the brightness function of luminosity $B(L)$. The most accurate data in this field is that relating to photometric sensibility and to persistence of vision.

Photometric sensibility is generally given as the ratio of the least perceptible increment to the total luminosity, $\delta L : L$. Call this $P(L)$. Measured in terms of a fixed unit of luminosity this increment would be LP. But sensibility is, properly speaking, the inverse of this, since the least perceptible increment is greater as sensibility is less. Sensibility to luminosity is then proportional to $1/PL$. Now the sensibility of an instrument at any part of the scale is the derivative of the scale reading with respect to the stimu-

lus measured, so that $S = dB : dL$. If then P can be determined as a function of L, B may be established.

Kŏnig and Brodhun's data[1] on photometric sensibility is very complete for six different wave-lengths and intensities ranging from 0.01 to 100,000 m.-c. This data is shown graphically in Fig. 3, where $\delta L : L$ is plotted against log nat. L. The dotted ordinates represent threshold values (L_0) of the luminosity. Since at the threshold value, the least perceptible light is the whole, $\delta L : L$ must there approach unity in value. The observed curves have been

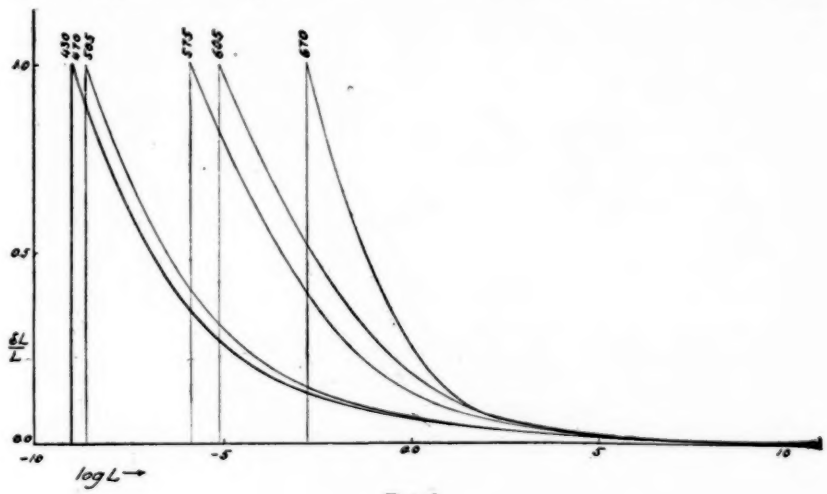

Fig. 3.

extrapolated to that value. At high intensities all the curves approach the same minimum constant value

$$P_m = 0.016.$$

Had the threshold value L_0 been used in each case as a unit of luminosity instead of a much larger common unit, all curves would be shifted to a common dotted ordinate and, execept the extreme red curve, would become congruent to far within the error of observation.

This form of curve may be represented by the simple function

$$P = a + be^{-ex},$$

where $x \equiv \log (L : L_0)$, $a \equiv P_m \mp 0.0016$, $b \equiv 1 - P_m \mp 0.984$ and

[1] König and Brodhun, l. c.

$bc \fallingdotseq 0.26 \pm 0.01$ (read from a large plot) when x is in natural logarithms. This would give for $P(L)$

$$P = P_m + (1 - P_m)(L_0 : L)^c.$$

Taking $S \equiv 1/PL$

$$S = \frac{1}{L(P_m + (1 - P_m)(L_0 : L)^c)}$$

$$\text{hence } B = \int S dL = \frac{1}{P_m c} \log (1 + P_m(L^c - L_0^c)),$$

the integration constant being zero since visual brightness must be zero for $L = L_0$ at the threshold value.

This gives then for the final form of the brightness function

$$B = B_0 \log (1 + P_m(L^c - L_0^c)),$$

or at moderate and large intensities, the simple form

$$B = a \log L + b.$$

Had we chosen a different variable T by which to measure the visual sensation, such that $B : B_0 \equiv \log T$, we should have for moderate and large intensities

$$L \fallingdotseq kT^4,$$

a very simple and suggestive relation.

The Purkinje Phenomenon is implied in several of the relations given above. Mathematically considered, this phenomenon involves the rate of change of luminosity or of visual brightness with energy. This should be greater for red and yellow than for blue and green. However it is demonstrated, the phenomenon rests ultimately upon the variation, $V(E)$, of visibility with intensity. If this variation did not exist there could be no Purkinje or other allied phenomenon.

The simplest demonstration is given by the ratio of two luminosities $L = EV$ and $L_0 = E_0 V_0$, the latter of which is very small. Then

$$\frac{L}{L_0} = \frac{E}{E_0} \frac{V}{V_0}.$$

But $V : V_0$ is always greater for red than for blue and green light (see Fig. 2) since the parameter a decreases and λ_m increases continuously with increase in E. Hence a given variation in E will

produce a much larger variation in L in the red than in the blue or green, judging by the data of Fig. 2, it might in some cases be 50 or 100 times as great. Taking instead of $L : L_0$, $dL : dE$ or $dB : dE$ leads after substitution to the same result.

Fechner's Law. — Fechner's law states that to produce a constant increment to a sensation requires a constant percentage increment to the stimulus. In this case the law will give $\delta L : L = \text{const.}$, where δL is the least perceptible luminous increment. For moderate and high intensities, this constant is about 0.016. But at the threshold of vision, evidently the least perceptible increment is the whole, hence $\delta L : L$ approaches the value unity and Fechner's law $\delta L : L = \text{const.}$ cannot hold at these low intensities.

To extend Fechner's law to cover these low intensities we must consider $\delta L : L = P$, the general photometric function discussed above. Hence, if the function $P(L)$ there constructed be correct, the complete form of Fechner's law is

$$\frac{\delta L}{L} = P_m + (1 - P_m)\left(\frac{L_0}{L}\right)^c$$

which is the simple general form

$$\frac{\delta S}{S} = a + bL^{-c}.$$

The Time Function.

Upon visual brightness as a function, $B(t)$, of the time depend persistence of vision and other allied phenomena. Since data in this field is of considerable accuracy and obtained with the simplest ap-

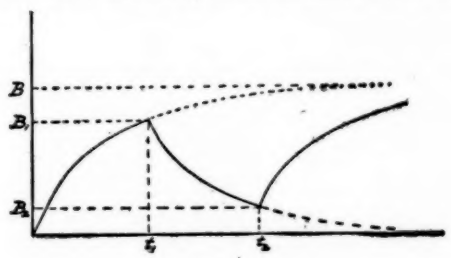

Fig. 4.

paratus, a function relating brightness to a time is of great interest. $B(t)$ we know to be of the general form shown in Fig. 4, starting

at zero, increasing rapidly at first and then less rapidly, and reaching a constant value after a short time.

$$B(t) = B(1 - \epsilon^{-at})$$

satisfies these conditions.

If now the eye be illuminated by light passing through a rotating sectored disk, $B(t)$ will be cut off after a time p_1, and again renewed after a further interval p_2 and so on. Let B_1 be the value attained by $B(t)$ after a period p_1, and B_2 the value if screened for a time p_2 after having attained the value B_1. Then

$$B_1 = B(1 - \epsilon^{-a_1 p_1})$$

$$B_2 = B_1 \epsilon^{-a_2 p_2}.$$

If the speed of the sector be varied until the sensation of flicker just vanishes, obviously

$$B_1 - B_2 = PL$$

the photometric constant measured as an absolute increment of luminosity. But

$$B_1 - B_2 = B(1 - \epsilon^{-a_1 p_1})(1 - \epsilon^{-a_2 p_2})$$

$$= B(1 - \epsilon^{-ap})^2$$

if $a_1 p_1 = a_2 p_2 \equiv ap$, that is if the segments cut from the disk equal those remaining and if initial rate of growth equals rate of decay of visual impressions. Hence, under these conditions

$$PL = B(1 - \epsilon^{a-p})^2$$

In the preceding work both P and B have been set up as functions of L and certain constants. Hence if L is a constant in any variation of conditions (say wave length) the critical frequency, p, must be constant as observed by Ferry.[1]

Substituting the values of P and B found above

$$(1 - \epsilon^{a-p})^2 = \frac{PL}{B} = \frac{P_m L + L_0^c(1 - P_m)L^{1-c}}{B_0 \log(1 + P_m(L^c - L_0^c))}$$

a form rather too complex to be useful as it stands. But the quantity in parenthesis is small so that its expansion in power series converges very rapidly. Further, the right hand member at

[1] E. S. Ferry, Am. J. Sc., 44, 192–207, 1892.

moderate and high intensities is approximately proportional to $L/(\log L + a)$, hence

$$a^2 p^2 = k \frac{L}{\log L + a}$$

is an approximate form of the above.

Luminous Efficiency.

It is the spectral integral of luminosity L to which the illuminating power and hence the commercial value of radiation is proportional. It may be well here to summarize the relations subsisting between these quantities and luminous efficiency.

Suppose the spectral *energy* $E(\lambda)$ of the radiation be given in watts per s.-m. per unit solid angle ω. From this suppose the spectal *luminosity* $L(\lambda)$ derived in arbitrary units (say *lumens*) per s.-m. per unit solid angle by means of the subsidiary visibility function $V(\lambda)$. Then

$$\text{Specific Emission} = \int_0^\infty E d\lambda \text{ in watts per unit solid angle.}$$

$$\text{Total Emission} = \int_s d\omega \int_0^\infty E d\lambda \text{ in watts.}$$

$$\text{Specific Illumination} = \int_0^\infty L d\lambda \text{ in lumens per unit solid angle.}$$

$$\text{Total Illumination} = \int_s d\omega \int_0^\infty L d\lambda \text{ in lumens.}$$

$$\text{Luminous Efficiency} = \frac{\text{total illumination}}{\text{total emission}} = \text{etc., lumens per watt.}$$

Since the space distribution of radiation and illumination must be identical, the space integrals may be dropped in calculating efficiency. And since $L = EV$.

$$\text{Luminous efficiency} = \frac{\int_0^\infty EV d\lambda}{\int_0^\infty E d\lambda}$$

the numerical value of which therefore depends upon the unit V_1 of visibility adopted in $V = V_1 e^{-a(\lambda - \lambda_m)^2}$ for $\lambda = \lambda_m$. This is the value assigned to the luminosity when the radiation is one watt per unit wave-length at $\lambda \equiv \lambda_m$.

Luminosity and Temperature. — If the spectral energy $E(\lambda)$ is known in functional form, say the Wien function, taking $V(\lambda)$ as derived gives the luminosity $L(\lambda)$ in terms of wave-length and certain parameters including temperature. The total luminosity then, which is the definite integral of $Ld\lambda$, is in the form of a function of these parameters. Unfortunately the Wien function itself leads to a form

$$L = EV = C_1 V_1 \lambda^{-5} e^{-c_2/\lambda T - a(\lambda - \lambda_m)^2}$$

which is very difficult or impossible to integrate. Once the integral of $Ld\lambda$ is obtained, visual brightness follows as a linear function of the logarithm of this integral.

Quality of Illumination. — Light is "white" when (*a*) $E(\lambda) =$ constant and (*b*) light of three or more colors is combined in certain proportions. According to the most widely accepted theories of color vision, the visibility curve $V(\lambda)$ is a composite of three elementary visibility curves such that

$$V = V_1 + V_2 + V_3.$$

These elementary curves appear to be indeterminate at present chiefly because there are more variables to be determined than equations to determine them. By assuming them of equal area (König and Dieterici) or equal maximum ordinate or of exactly the same form they may be deduced by means of the observed sensibility of the eye to slight color differences. Assuming them to be known, and taking $L_1 = EV_1$, etc., evidently the criterion for white light is that

$$\int L_1 d\lambda : \int L_2 d\lambda : \int L_3 d\lambda = \int V_1 d\lambda : \int V_2 d\lambda : \int V_3 d\lambda,$$

whatever the form of the spectral energy curve $E(\lambda)$.

SUMMARY.

1. Radiation $E(\lambda)$ may be translated into light $L(\lambda)$ by means of the relation $L = EV$ where $V(\lambda)$ is the visibility function of the simple form

$$V = V_1 e^{-a(\lambda - \lambda_m)^2}.$$

The parameter V_1 is arbitrary while for low intensities $a = 4$ and $\lambda_m = 5.1$ s.-m. a decreases and λ_m increases steadily with increasing intensity, reaching the values $a = 2$ and $\lambda_m = 5.6$ at an intensity of about 100 m. c.

2. The least perceptible percentage differences in luminosity to which the eye is susceptible is closely represented by the function

$$P(L) = P_m + (1 - P_m)(L_0 : L)^c,$$

where $P_m \mathrel{\dot{=}} 0.016$ and $c \mathrel{\dot{=}} 0.26$ and L_0 is the threshold value of the luminosity. P_m and c are nearly if not quite independent of the individual and of the color.

3. Fechner's law $\delta L : L =$ constant, holding for moderate and high intensities, may be extended to cover low intensities as well if written in the form

$$\frac{\delta L}{L} = P_m + \frac{1 - P_m}{(L : L_0)^c}.$$

4. Visual brightness or apparent luminosity appears to be a function of the form

$$B = B_0 \log (1 + P_m(L^c - L_0^c)).$$

No attempt has been made to derive these functions analytically from physiological data, such attempts would but obscure the points at issue. And as the methods of synthetic function theory are as yet largely those of cut-and-try, the functions here obtained are little more than tentative at best. Still they are vastly more than empirical formulæ and of more value than any equations derived from largely hypothetical physiological data.

My object has been not so much to arrive at the final form of these functions as to show the essential relations between them. When better data, covering a wider range and more individuals has been obtained, the fundamental functions and their parameters may be more firmly established.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
November, 1906.

STANDARD CELLS.

BY K. E. GUTHE AND C. L. VON ENDE.

1. According to recent investigations electrolytically prepared mercurous sulfate seems to be superior to chemically prepared sulfate as the depolarizer in standard cells[1] and the first results of a series of observations on cadmium cells[2] containing this substance showed for a period covering several months a remarkable agreement of their electromotive forces. For this reason one of the authors made use of Clark and cadmium cells with electrolytic mercurous sulfate for the determination of the electromotive forces of standard cells by absolute measurement.[3]

These cells have been transported from Washington to Iowa City, being carefully protected against mechanical disturbances. During the past winter they stood, however, for some weeks in diffused light and were exposed for a few days to large temperature variations. A comparison of the cells after this time showed that the electromotive force of the cadmium cells had decreased relatively with respect to that of the Clark cells, but accurate measurements could not be made at that time on account of lack of sufficiently sensitive instruments. The comparisons reported on in this paper were made with a calibrated Wolff potentiometer and a galvanometer allowing with certainty a reading to the fifth decimal place.

The cells were placed in a thermostat whose temperature could be kept constant at 25° C. within 0.02 degrees. The temperature was measured by means of a thermometer subdivided into fiftieths of a degree and calibrated at the Bureau of Standards. Table I. shows the relative change of the electromotive forces of the cells in the interval of a year. The R series are Clark cells with electrolytic mercurous sulfate, C_3 and K_{10} cadmium cells with chemically pre-

[1] Wolff, Trans. Amer. Electroch. Soc., 5, 49, 1904. Carhart and Hulett, ibid., 5, 59, 1904. Hulett, Zeitschr. f. physik, Chem., 49, 483, 1904; PHYS. REV., 22, 321, 1906. Compare also: Reichsanstaldt, Zeitschr. f. Instr. kunde, 26, 120, 1906.

[2] Carhart, Trans. Internat. Electr. Congr. St. Louis, vol. 2, 125, 1904.

[3] Guthe, Bull. Bur. Standards, vol. 2, 33, 1906.

pared sulfate and the rest cadmium cells with the electrolytic material. More detailed information as to the construction of these cells may be found in the Bulletin of the Bureau of Standards, vol. 2, p. 60. Some of the cells have been set up by Professor Carhart, some by Professor Hulett. As reference value in the following table the electromotive force of the Clark cell as determined by absolute measurement was chosen, namely, 1.42040 volts at 25° C.

TABLE I.

Old Cells.

	Date of Construction.	September 17, 1905.	October 15, 1906.	November 28, 1906.	January 12, 1907.
R_2	May 1904.	1.42040	1.42040	1.42040	1.42040
R_4		40	40	40	41
C_3	Nov. 1903.	1.01857	1.01842	1.01844	1.01842
E_3	Jan. 1904.	31	04	01	1789
F_8	Feb. 1904.	27	19	22	1822
F_9		28	08	07	1799
K_6	July 1904	33	24	25	1826
K_{10}		33	11	12	02
O_1	Feb. 1905.	33	23	23	24
O_2		33	19	22	21

The change in most of the cadmium cells amounts to about 0.0001 volt. Since the potentiometer was carefully calibrated and the thermometer correction known, this change is larger than the possible error in the apparatus. The relative values of F_8, K_6 O_1 and O_2 have remained about the same as last year, but for the others the decrease has been larger and quite irregular.

2. To decide the question if a change had taken place in the Clark cells chosen as reference standards it became necessary to construct new standard cells. At the same time we wished to ascertain how closely such cells would agree in electromotive force if constructed by different observers and with material from different sources. No such comparative investigation has yet been published so far as we know.

Professor Hulett kindly sent us some of his electrolytic mercurous sulphate and we prepared two different lots following closely the method published by him. We shall designate these two as lot 1 and 2 respectively. The sulfate was made from pure mercury in an

acid prepared by mixing one part of concentrated sulfuric acid of specific gravity 1.84 with six times its volume of distilled water. The current density was between two and three amperes per 100 square centimeters. The first lot consisted of very fine crystals; the second was stirred longer after the breaking of the current and was therefore coarser.

The sulfate was filtered in a Gooch crucible and after the acid had been thoroughly drained off, was washed repeatedly with alcohol and saturated zinc or cadmium sulfate solution. We found it necessary to be very careful in washing, especially with the finest crystals. After the last washing with the salt solution the top layer was removed and the paste prepared in the usual manner.

The mercury was either distilled by Hulett's method[1] or distilled twice in vacuo in an ordinary mercury still.

The zinc amalgam was prepared in the usual way by dissolving zinc in mercury. In some of the cadmium cells we used an amalgam prepared electrolytically.

CLARK CELLS.

3. Two series were set up, the *a* and *b* series. In the *a* series the zink amalgam contained 9 per cent. of zinc. Our mercurous sulfate No. 1 was used in the *a* series. In the *b* series the amalgam contained 10 per cent. of zinc, and Hulett's mercurous sulfate was used. Mercury distilled by Hulett's method was used in cells a_1, a_2, b_1 and b_2, the other mercury in a_3 and b_3. The zinc sulfate crystals were recrystallized from a neutralized solution of Schuchardt's c. p. sulfate and the solution in the cells made from these crystals.

In table II, the electromotive forces as determined on different days are tabulated. The values for the cells with the very fine mercurous sulfate were at first somewhat low, but increased rather suddenly between the ninth and fourteenth day, then slower until they reach a value a little higher, but still agreeing well with that of the old Clark cells. The *b* cells, the mercurous sulfate of which had not been as difficult to wash, had immediately after construction a value agreeing within 0.00003 volt with that of the old cells and remained constant ever since.

[1] Hulett and Minchin, PHYS. REV., 21, 388, 1905.

The great difference in the size of the mercurous sulfate crystals had apparently no great influence upon the final value of the electromotive force; the two different samples of mercury behaved electrolytically exactly alike.

TABLE II.

Clark Cells.

	Date of Construction.	Oct. 14, 1906.	Oct. 20, 1906.	Oct. 26, 1906.	Nov. 6, 1906.	Nov. 20, 1906.
a_1		1.42025	1.42029	1.42037	1.42041	1.42043
a_2	Oct. 13, 1906	—	—	—	40	43
a_3		24	30	38	42	43
b_1			38	39	38	38
b_2	Oct. 19, 1906		37	40	39	38
b_3			37	39	38	38

	Date of Construction.	Dec. 5, 1906.	Dec. 24, 1906.	Jan. 19, 1907.
a_1		1.42044	1.42045	1.42045
a_2	Oct. 13, 1906	44	45	45
a_3		45	44	45
b_1		38	38	—
b_2	Oct. 19, 1906	38	38	38
b_3		37	38	38

We also prepared some cells in which the directions were not followed very closely. In a_4, a_5, and a_6 the mercurous sulfate (Hulett's) was not thoroughly freed from alcohol which produced a lowering of the electromotive force by over 0.00040 volt without an appreciable change in course of time. Cells b_4 and b_5 were prepared with our mercurous sulfate No. 1, thoroughly washed but it was mixed with about the same bulk of zinc sulfate crystals and the paste made very thin. These cells had from the start an electromotive force 0.00015 volt higher than the normal value and remained perfectly constant.

It is apparent that the treatment of the mercurous sulfate plays a much more important rôle than the sulfate in itself. a_1, a_2 and a_3 as well as b_4 and b_5 were made with our sulfate, but the former cells show quite a different behavior from that of the latter; the same is true of the cells in which Hulett's sulfate was used.

After a few weeks a_3 a_4 and a_5 as well as b_4 and b_5 were placed for three hours on a circuit of 1,000 ohms external resistance Nov. 4 and Nov. 23 respectively. It was hoped that, if free acid had

TABLE III.

Clark Cells.

	Date of Construction.	Oct. 14, 1906.	Oct. 18, 1906.	Oct. 20, 1906.	Oct. 26, 1906.	Nov. 6, 1906.
a_4	Oct. 13, 1906	1.41968	1.41992	1.41992	1.41994	1.41982
a_5		80	1.42004	1.42003	99	80
a_6		87	06	05	1.42005	93
b_4	Oct. 19, 1906			1.42056	1.42057	1.42056
b_5				56	57	56

	Date of Construction.	Nov. 23, 1906.	Dec. 5, 1906.	Dec. 24, 1906.	Jan. 19, 1907.
a_4	Oct. 13, 1906	1.41980	1.41981	1.41980	1.41980
a_5		80	81	81	80
a_6		91	91	91	91
b_4	Oct. 19, 1906	1.42053	1.42054	1.42055	1.42054
b_5		53	54	54	54

been present, this treatment would change the electromotive force. After opening the circuit the electromotive force had dropped about 0.00040 volt, but it returned in about two hours to about its original value and remained constant after that.

CADMIUM CELLS.

4. The cadmium sulfate crystals were obtained from three different solutions; of these *A* was made from Merck's pure cadmium sulfate, *B* from Baker's c. p. material, and *C* from mixed turbid crystals picked out from either *A* or *B*. In the *c* series lot 2 of our electrolytic mercurous sulfate washed with alcohol and saturated cadmium sulfate solution (made from selected clear crystals) was used. The amalgam contained 12.5 per cent. cadmium and was made by dissolving Kahlbaum's cadmium (käufflich) in warm mercury.

Since it is rather troublesome to obtain a sufficient number of perfectly clear crystals from a small bulk of cadmium sulfate solution, we used also somewhat turbid crystals. The crystals in c_1 were selected large clear crystals from solution *C*, in c_2 small selected, somewhat turbid crystals from solution *B*, and in c_3 large selected crystals somewhat cloudy, from *A* and *C*.

In the *d* series Hulett's mercurous sulfate was used, treated the same way as ours in the *c* series. In d_1 and d_2 selected clear crys-

tals were used from solution C, in d_3 and d_4 small crystals which had been formed at the bottom of the crystallizing dish from solution A and which were washed a few times with distilled water, after the mother liquor had been filtered off. No attempt was made to select the crystals for these cells. The solution in the cells was always made from the crystals used in filling. d_1 and d_3 contain the same amalgam as the c cells, d_2 and d_4 electrolytically prepared amalgam.

Table IV. shows that the cadmium cells c gave at first a value for the electromotive force of nearly 1.01840 volt. which dropped, however, within a few days by about 0.0001 volt. and remained fairly constant after that. The drop in the d cells is, however, considerably larger than in the c cells, and especially pronounced in d_3 and d_4. The different methods of preparing the amalgam does not seem to have an effect upon the electromotive force.[1]

TABLE IV.

Cadmium Cells.

	Date of Construction.	Nov. 3, 1906.	Nov. 6, 1906.	Nov. 10, 1906.	Nov. 12, 1906.	Nov. 17, 1906.
c_1		1.01840	1.01838	1.01836	1.01835	1.01835
c_2	Nov. 2, 1906	38	36	35	34	35
c_3		39	37	36	33	33
d_1				1.01842	1.01840	1.01834
d_2				42	42	37
d_3	Nov. 9, 1906			46	40	28
d_4				47	45	35

	Date of Construction.	Nov. 23, 1906.	Nov. 28, 1906.	Dec. 5, 1906.	Dec. 13, 1906.	Jan. 19, 1907.
c_1		1.01834	1.01833	1.01831	1.01829	
c_2	Nov. 2, 1906	35	34	32	30	1.01829
c_3		32	32	31	28	28
d_1		1.01827	1.01823	1.01815	1.01811	1.01805
d_2		33	31	24	19	13
d_3	Nov. 9. 1906	19	14	10	06	03
d_4		24	19	11	07	02

[1] Towards the end of December, 1906, we exchanged Clark cell b_1 and cadmium cell c_1 for a Clark and cadmium cell constructed by Dr. Wolff at the Bureau of Standards. Wolff's Clark cell has an electromotive force 0.00006 volt. lower, and his cadmium cell an electromotive force 0.00003 volt. lower than ours of the same type. Dr. Wolff writes that he has found practically the same difference in his comparisons at the Bureau which shows that the cells have not been affected by transportation.

5. In a general way this investigation corroborates Hulett's recently published results,[1] that Clark cells containing electrolytic mercurous sulfate when set up with proper precautions will have an electromotive force of 1.42040 volts at 25° C. within a few one-hundred thousandths of a volt and remain constant in course of time. The cadmium cells show immediately after the setting up of an electromotive force of 1.0184 volts, closely agreeing with the value given by Hulett.

There is, however, one notable difference. Hulett's *A* cells, set up October 21, 1905, have an electromotive force of 1.01843 volts and have kept this value for at least six months, later results not having been published. His old cells (*F* series) are now about 0.00011 volt lower. Hulett, therefore, drew the conclusion that in the course of time the electromotive force slowly decreases and described a number of very interesting experiments which seem to show that the system in the cathode leg of the cells is not in equilibrium. But the *F* cells, though originally of the same high value, decreased to their present low value within a week, and have remained, with one exception, constant since then. A similar rapid decrease is noticeable in the *J* and *K* cells on which Professor Carhart[2] has reported. Hulett explains this difference in behavior by a difference in the depth of the paste. We have made the paste in the cells over two centimeters deep, but nevertheless the initial decrease is rapid. This difference in the behavior of Hulett's and our cells can hardly be due to the mercurous sulfate and we are at a loss to explain it unless it is to be found in the unstable equilibrium between the cadmium sulfate solution and the rest of the material at the cathode or in a difference in the method of construction of which we are not aware. Our results do not extend over a sufficiently long period to decide the question whether or not there will be a continuous slow decrease of the electromotive force in course of time.

After the satisfactory results obtained with the Clark cells the apparent difficulty for different observers to set up cadmium cells, which remains constant from the start and are practically identical

[1] Hulett, PHYS. REV., 23, 166, 1906.
[2] Carhart, l. c.

in their electrical behavior, seems to indicate that there is still some factor of uncertainty in the method of construction for the latter and this demands further investigation.

The large changes in the electromotive forces of the old cadmium cells mentioned above, are, possibly, not due to a continuous gradual decrease in course of time but to sudden changes brought about by the severe treatment to which they were subjected. It is however very gratifying to know that the causes for such changes have had no influence whatever upon the electromotive force of the Clark cells, since the latter agree so closely with the newly prepared cells.

Conclusions.

1. The electromotive forces of standard cells set up with electrolytic mercurous sulfate by different observers and from different sources agree under otherwise perfectly equal conditions within a few one hundred-thousands of a volt.

2. Mercury twice distilled in an ordinary vacuum still gives the same results as mercury distilled by Hulett's method.

3. Amalgam prepared in the usual way gives the same results as electrolytic amalgam.

4. Clear and cloudy cadmium sulfate crystals produce no difference in the electromotive force of the cadmium cells.

5. Clark cells with electrolytic, coarsely grained mercurous sulfate have immediately after construction an electromotive force of 1.4204 volts at 25° C. within a few one hundred-thousandths of a volt and remain constant in course of time.

6. We have been unable to construct cadmium cells which do not show an identical decrease in electromotive force.

The State University of Iowa,
Iowa City, January, 1907.

AN OPTICAL DEVICE FOR DEFLECTION INSTRUMENTS.

BY EDWIN F. NORTHRUP.

THE system of a suspended instrument, as a galvanometer, when subjected to vibrations will have a motion that is compounded of a rotation about a vertical axis and a rotation about a horizontal axis. If the system is held between an upper and a lower filament that are under tension, the horizontal component of the vibratory motion may be practically eliminated by accurately balancing the system about the line of the suspension. Sudden horizontal movements will then have little or no tendency to produce rotations.

Furthermore, in d'Arsonval galvanometers the electromagnetic damping forces quickly reduce to rest a system started into oscillation about a vertical axis. Oscillations of the system about a horizontal axis are not damped out by electromagnetic forces and are more difficult to eliminate by balancing or weighing the system. The vertical movements of the image of the scale seen in a telescope blurr the image, and often are so bad as to make it impossible to read the scale. To use suspended instruments in places subject to considerable vibration, the julius suspension is successfully employed, but this apparatus is both costly and cumbersome.

I give here an optical device which I have devised and employed that *entirely* eliminates the vertical component of the movement of the image of the scale. This device used in conjunction with a well-balanced system enables the scale to be easily read when the system itself is in violent vibration about a horizontal axis.

The device is clearly explained by reference to the diagram. Here *B* is a side elevation of a D'Arsonval coil fitted with two mirrors, their faces making an angle of 90 degrees with each other. *A* is a front elevation of the same.

The course of the ray from scale to mirrors and thence to tele-

scope is shown in dotted line. It is easily seen that rotation of the ninety-degree-mirror system about a horizontal axis does not change the direction of the light ray from scale to telescope. Furthermore, if the scale and telescope are vertically raised or lowered together in an arc of which the mirror is the center the image of the scale continues visible in the telescope. It is therefore very easy "to find the scale" as the necessity for the up and down

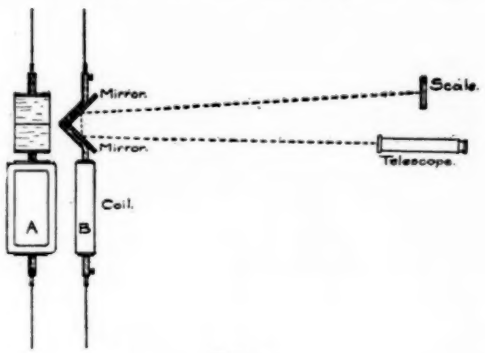

Fig. 1.

adjustment of telescope and scale when made to move together is dispensed with.

The loss of light by the double reflection is not noticeable and the double mirror system operates for deflections about a vertical axis in all respects like a single plane mirror.

If it is required to cast a spot of light on a scale this may be effected by placing a plano-convex lens of suitable focal length, in front of the mirror system.

THEORY OF THE ELECTRODELESS RING-DISCHARGE.

A CORRECTION.

BY BERGEN DAVIS.

PROFESSOR R. WACHSMUTH, of Berlin, has kindly written me a letter pointing out a lack of clearness and an error in a paper with the above title published in the PHYSICAL REVIEW, March, 1905.

The method there developed of finding the electrical intensity at a point within a flat spiral coil carrying a varying current is not free from criticism, and I take this occasion to give a more complete and simpler solution of the problem.

The final value of the electrical intensity found by the corrected method does not differ materially from the value previously found, and hence does not affect the constants of the discharge as calculated by means of the theory.

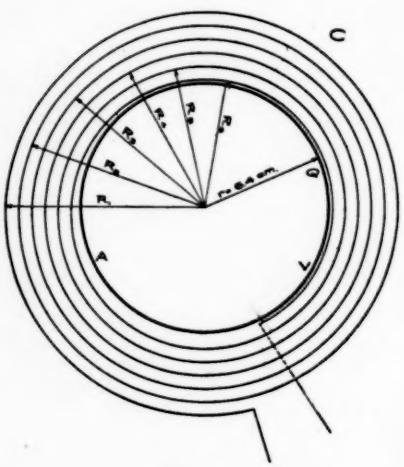

Fig. 1.

The problem is to find the electrical intensity X acting at the inner wall of the vessel AQ when an oscillating current γ flows through the coil C.

The coil consisted of six turns of wire wound in a flat spiral. It was connected to the outer coatings of two Leyden jars, the inner coatings of which were connected to a static machine and a spark gap.

On the passage of a spark a current flowed through the coil, which may be represented by

$$\gamma = \frac{CP}{\sqrt{LC}} \sin \frac{t}{\sqrt{LC}}$$

if the damping is neglected.

C is the capacity of the Leyden jars, L the self-induction of the system

and P is the potential to which the jars were charged at the instant of the passage of the spark.

The mean radii of the separate turns of the coil were:

$R_1 = 9.3$ cm. $\quad R_4 = 8.58$ cm.

$R_2 = 9$ $\quad R_5 = 8.34$

$R_3 = 8.82$ $\quad R_6 = 8.1$

The inner radius of the vessel was $r = 6.4$ cm.

It is desired to find the electrical intensity X acting along the inner wall of the vessel AQ.

At any point within the coil

$$\text{Curl } E = -\frac{\partial H}{\partial t}. \qquad (1)$$

Where E refers to the electrical intensity arising from the varying magnetic field due to the current γ in *one* of the turns of the coil.

Then along the wall of the vessel AQ whose length is $l = 2\pi r$, by Stokes' theorem we have

$$\int_0^{2\pi r} Edl = \iint \text{curl } Eds$$

or

$$E = -\frac{1}{2\pi r}\frac{\partial}{\partial t}\iint Hds. \qquad (2)$$

$\iint Hds$ is the total magnetic flux through the circle AQ due to the current in *one* of the turns of the coil. It is also equal to the coefficient of mutual induction M between the circle AQ and one of the turns of the coil multiplied by the current γ. That is

$$\iint Hds = M\gamma.$$

Introducing this in equation (2) together with the expression for γ and differentiating

$$E = -\frac{1}{2\pi r}\frac{P}{L}\cos\frac{t}{\sqrt{LC}}M.$$

The mean value of which is

$$E = -\frac{1}{\pi^2 r}\frac{P}{L}M. \qquad (3)$$

For the six turns of the coil (3) becomes

$$X = E_1 + E_2 + \cdots E_6 = -\frac{1}{\pi^2 r}\frac{P}{L}\sum_1^6 M. \qquad (4)$$

The coefficient of mutual induction between a turn of wire and a circle AQ is given by Maxwell (Elect. & Mag., Vol. II., p. 345).

It may be expressed in the terms of this coil and summed as follows:

$$\sum_{R=1}^{R=6} M = \sum_{R=1}^{R=6} 4\pi r \log\frac{8r}{R-r}\left(1+\tfrac{1}{2}\frac{R-r}{r}+\tfrac{1}{16}\left(\frac{R-r}{r}\right)^2-\tfrac{1}{32}\left(\frac{R-r}{r}\right)^3+\cdots\right)$$
$$+\sum_{R=1}^{R=6} 4\pi r\left(-2-\tfrac{1}{2}\frac{R-r}{r}+\tfrac{3}{16}\left(\frac{R-r}{r}\right)^2-\tfrac{1}{48}\left(\frac{R-r}{r}\right)^3+\cdots\right).$$

Where R has the values R_1, $R_2 \cdots R_6$ and r is the radius of the circle AQ.

Solving the above expression for the various values of R, equation (4) reduces to

$$\frac{X}{P} = .0017.$$

This is the ratio of the electrical force acting on the gas at the inner wall of the vessel to the potential at which the spark passes.

This value of X/P differs but little from the value given in my previous paper which was

$$\frac{X}{P} = .00165.$$

This correction is less than the experimental errors, consequently it will make no material change in the constants of the discharge calculated by means of the theory from the experimental data.

PHŒNIX PHYSICAL LABORATORY,
COLUMBIA UNIVERSITY,
October 6, 1906.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE THIRTY-FIFTH MEETING.

THE annual meeting of the Physical Society was held in Fayerweather Hall, Columbia University, New York City, December 27–29, 1906.

The business meeting of the Society was held Friday morning December 28, President Barus being in the chair. Tellers having been duly appointed the Society proceeded to the election of officers for the year 1907. The following officers were elected:

President. — Edward L. Nichols.

Vice-President. — Henry Crew.

Secretary. — Ernest Merritt.

Treasurer. — William Hallock.

Members of the Council. — H. A. Bumstead and C. E. Mendenhall.

All the other sessions of the Society were joint sessions with Section B of the American Association for the Advancement of Science. President Barus occupied the chair during the morning session of December 28 and both sessions of December 29, while Vice-President Sabine of Section B presided for the session of December 27 and the afternoon session of December 28.

The following papers were presented.

1. A Colorimetric Determination of the Relative Humidity of the Air. HENRY E. WETHERILL.

2. The Kinetic Theory of Gases. LUIGI D'AURIA.

3. The Effect of a Magnetic Field upon the so-called Spontaneous Ionization of Air. W. W. STRONG.

4. The Standard Cell. F. A. WOLFF and C. E. WATERS.

5. The Equilibrium of Mercurous Sulphate and Mercury and Cadmium Sulphate. F. A. WOLFF and C. E. WATERS.

6. Projections of the Sphere Appropriate for Studying the Dynamical Problems of the Atmosphere. (Read by title.) CLEVELAND ABBE.

7. Final Report on Ether Drift Experiments. E. W. MORLEY and D. C. MILLER.

8. The Compensated Two-circuit Electrodynamometer for Alternating Current Measurements of Precision. EDWARD B. ROSA.

9. The Power Factor and Temperature Coefficient of Mica Condensers. E. R. ROSA and F. W. GROVER.

10. The Transmission of Roentgen Rays through Metallic Sheets. J. M. ADAMS.

11. The Transformation into an Electric Current of Radiation Incident on a Moving Surface. BERGEN DAVIS.

12. Energy necessary to Ionize a Molecule by Impact of Negative Electrons. BERGEN DAVIS.

13. The Influence of Electrical Fields upon Spectral Lines. G. F. HULL.

14. Production of Radium by Actinium. BERTRAM B. BOLTWOOD.

15. Production of Radium from Actinium. E. RUTHERFORD.

16. On the Conductivity of the Air caused by Certain Compounds during Temperature Change. FANNY C. GATES.

17. Fluorescence Absorption. E. L. NICHOLS and ERNEST MERRITT.

18. The Absorption of Some Solids for Light of Short Wave-length. THEODORE LYMAN.

19. The Distribution of the Energy emitted by a Righi Vibrator. C. R. FOUNTAIN and F. C. BLAKE.

20. Diffraction of Electric Waves of Short Wave-length. A. D. COLE.

21. The Constants in Gas-Viscosity. WILLARD J. FISHER.

22. On the Variation of the Heat of Mixture with Concentration and Temperature. BERTHA M. CLARK.

23. A Study of the Reversible Pendulum. JOHN C. SHEDD and JAMES A. BIRCHBY.

24. On the Nature of Optical Images. ALBERT B. PORTER.

25. The Mutual Inductance of two Coaxial Coils. E. B. ROSA.

26. The Mutual Inductance of a Circle and a Coaxial Helix. The Lorenz Experiment and the Ayrton-Jones Absolute Electro-dynamometer. E. B. ROSA.

27. Mutual Inductances for Laboratory use. E. B. ROSA.

28. A Color Mixer. ALBERT B. PORTER.

29. Wave-Metrical Measurements with Wireless Telegraph Circuits. (Read by title.) G. W. PIERCE.

30. Geometrical Theory of Radiating Surfaces with Discussion of Light Tubes. EDWARD P. HYDE.

31. The Hissing Point of the Metallic Arc. W. G. CADY.

32. On the Determination of Specific Inductive Capacities by the Gold Leaf Electroscope. (Read by title.) J. C. MCLENNAN.

33. Some Notes on the Magnetic Properties of Heusler's Alloys. J. C. MCLENNAN.

34. On the Magnetic Susceptibility of Mixtures of Salt Solutions. J. C. McLennan.

35. Helion, a new Incandescent Lamp Filament. H. C. Parker and W. G. Clark.

36. The Magnetic Rotation of Na Vapor at the D Lines. R. W. Wood.

37. Fluorescence Spectra of Mercury Vapor. R. W. Wood.

38. Hydraulic Analogy of the Welsbach Mantle and other Radiators. R. W. Wood.

39. The Shielding of a Highly Sensitive Galvanometer. E. F. Nichols and S. R. Williams.

40. On Magnetic Shielding. A. P. Wills.

41. On the Temperature of the Mercury Arc. Chas. T. Knipp.

42. Remarkable Optical Properties of Carborundum. Lewis E. Jewell.

43. Models illustrating the Motion of a Violin String. Harvey N. Davis.

44. The Motion of a Violin String under light Bowing. Harvey N. Davis.

45. Distributions of Ions and of Nuclei in Dust Free Air. (Read by Title.) Carl Barus.

Adjourned Saturday afternoon, December 29, 1906.

Ernest Merritt,
Secretary.

On Effects of the Electrical Discharge on the Acetylene Flame.[1]

By C. F. Lorenz.

THE observation that the brightness and whiteness of the light emitted by a flame of ordinary illuminating gas was increased slightly by the passage through it of the discharge from an induction-coil, the primary of which was fed with alternating current, led to experiments with various carbon containing flames in order to bring out this effect in a more pronounced manner. It was found that in the case of a cylindrical jet of acetylene issuing under considerable pressure from a small hole the light could be increased about five-fold. The formation of soot, at first troublesome, was avoided by placing the pointed brass electrodes in the invisible mantle of vapor surrounding the flame out of contact with the luminous part, one being placed near the base and the other near the top or middle of the flame on the opposite side. Simultaneous measurements of current, candle-power and voltage were made on a flame having an initial

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

candle-power of 7.5, flame height of 59 mm. and electrode distance of 24 mm., the rate of flow of gas being .34 cubic feet per hour. There was a linear increase of candle-power with current up to a limiting value of 38.0, the current being then 45 milliamperes and the difference of potential between the electrodes 2,700 volts. The difference of potential first increased with increase of current, as in the case of an ordinary resistance and then decreased as in the case of an arc, the maximum value, amounting to 3,000 volts occurring when the current was about 10 milliamperes, this being long before the beginning of appreciable arcing at the electrodes.

Observation of the flame in a rotating mirror shows that it shortens and brightens each time the discharge passes. The flame, of course, emits a humming sound. On employing direct current, obtained from a number of small generators in series, an almost noiseless steady light resulted. Several more or less useful applications of such a current carrying acetylene flame, which gives a white light strong in the violet end of the spectrum, suggest themselves. For example, it may be applied in cases in which it is desired to produce rapid variations in the intensity of a light by means of small currents, such as microphone currents; the secondary of an induction coil, which may serve at the same time as as a steadying resistance, is put in series with the flame and the variations or interruptions made in a circuit containing the primary.

THE RADIATION CONSTANTS AND TEMPERATURE OF THE NERNST GLOWER.[1]

BY C. E. MENDENHALL AND L. R. INGERSOLL.

FROM a spectrophotometric study of a Nernst glower at several known temperatures (the melting-points of gold, palladium and platinum) the constants in the isochromatic radiation formula have been determined, on the assumption that the equation is of Wien's form: $\log E = K_1 + K_2 1/T$. This establishes a radiation-temperature scale for the glower, and by exterpolation the temperature of the glower with normal or any desired power consumption can be estimated. Experience has shown that the isochromatic radiation function gives about the most reliable method of estimating extremely high temperatures. The normal temperature is found to be about 2,300 abs., disagreeing with the recent determination of Hartman, 1,800° abs., which can readily be shown to be too low. The use of a Nernst filament as a meldometer for determining very high melting points is also discussed, as also the relative advantages of radiation and thermoelectric methods for determining flame and other high temperatures.

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

Change of Phase Due to the Passage of Electric Waves Through Thin Films, and the Index of Refraction of Water for Such Waves.[1]

By Wm. R. Blair.

I. The apparatus for the production of the waves consists of a high frequency device and a finely adjustable gap in oil. The interferometer is of the Michelson type, paraboloidal reflectors being used about the gap and the receiver. Two receivers are used, one receiving the direct radiation from the gap, and the other the energy of the interfering waves. They are connected with the galvanometer so as to oppose each other. The time during which the current passes through the galvanometer is the same for all readings, being controlled by a pendulum.

II. It is found that the change of phase is not a linear function of the thickness of the film used.

III. These results seem to make it necessary to modify the conclusions drawn from previous experimental work upon thin wedges of absorbing media.

On the Magnetic Behavior of Certain Alloys of Nickel.[1]

By Bruce V. Hill.

MANY nickel alloys are of great importance. Such are the nickel steels and a number of alloys used as resistances. The thermal, mechanical and electric properties seem to be closely connected with the magnetic. These relations may be accounted for by the allotropic theory.

Since the depression of the transformation point of nickel in an alloy is very like that of the freezing point of a solvent by a solute it was sought to find whether the formula of van't Hoff

$$\Delta t = .02\, T^2 a / ML$$

does not apply to the case of alloys. Alloys of nickel with copper, tin and silver were studied and the results are tabulated together with those of other observers upon other alloys of nickel. It is seen that the depression of the point of transformation of the nickel is not inversely proportional to the atomic weight and no simple relation has been found.

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

THE VISCOSITY OF WATER AT LOW RATES OF SHEAR.[1]

BY LAWRENCE E. GURNEY.

BECAUSE of its bearing on the theory of fluids an attempt was made to measure the coefficient of viscosity of water at much lower rates of shear than have been used hitherto. A ring of water 1 cm. thick, 5 cm. in radius and about 5 cm. in effective length was enclosed between concentric brass cylinders; the outer one rotating at uniform speed — the inner one hung upon a small steel wire. End effects and the constants of the suspension were eliminated experimentally. Three rates of shear were used of about 5, 1.3, and .6 radians per second. The results of these three series were self-consistent within .3 per cent. and within experimental error appeared to agree with the values obtained at ordinary rates of shear.

Prof. A. W. Duff (Phil. Mag., May, 1905) describes experiments from which he concludes that the soluble constituents of glass may enormously increase the viscosity coefficient of water at low rates of shear. This conclusion was tested by the method given above and was not confirmed.

Incidental observations were made showing that under certain conditions the surface film of water may behave like a rigid body.

ON DISTRIBUTIONS OF NUCLEI AND IONS IN DUST-FREE AIR.[2]

BY CARL BARUS.

I HAVE recently found it desirable to gather my data together for comparison. There is in fact a serious discrepancy between Mr. C. T. R. Wilson's results and mine when reduced to the same scale. Mr. Wilson's supersaturations for negative ions and cloud are distinctly higher, which certainly can not mean that my fog chamber is in these regions inferior to his own. Thus in moderately ionized dust-free air my condensations begin at a drop of about 18.5 cm. from 76 cm. as compared with 20.5 in Wilson's apparatus; similarly my fogs begin at the drop 20.3, Wilson's at 27.7. Furthermore at low ionization even the vapor nuclei of dust-free wet air become efficient in the presence of ions. It seems impossible therefore that any positive ions should fail of capture.

Notation. — The whole case may best be represented graphically. In my apparatus, however, the adiabatic volume expansion v_1/v is a trouble-

[1] Abstract of a paper presented at the Chicago meeting of the Physical Society, December 1, 1906.

[2] Read by title at the New York meeting of the Physical Society, December.

some datum to compute; it appears as

$$\frac{v_1}{v} = \frac{(p-\pi)\left(\frac{p'-\pi'}{p-\pi}\right)^{\frac{k-c}{k}}\left(1-\frac{\pi_1'-\pi_1}{p_1-\pi_1}\right)^{c/k}+\frac{v''}{V}}{(p_3-\pi)\left(1+\frac{v''}{V}\right)},$$

where p and p' are the pressures in the fog and vacuum chambers before exhaustion, p_3 their common pressure when in communication after exhaustion, always at the same temperature. p_3 is very nearly the adiabatic pressure. The volume ratios of the chambers is $v''/V = .064$; the π's denote the different vapor pressures and k and c the specific heats. With a large vacuum chamber the approximation

$$v_1/v = [(p-\pi)/(p_3-\pi_1)]^{c/k},$$

suffices, so that if $\delta p = p - p_3$, the convenient variable for the comparison of exhaustions is the relative drop $[\delta p - (\pi - \pi_1)]/(p-\pi)$. This is used in the diagram with the approximate equivalent of v_1/v.

Graphs. Dust-free Air. — The chart contains a number of curves, showing the nucleation in thousands per cubic centimeter (computed from the angular diameter of coronas) in terms of the exhaustion. Typical cases are given, in their lower parts only. Thus the curve for the vapor nuclei of dust-free air begins appreciably below .25 ($v_1/v = 1.22$; adiabatic drop from 76 cm., 20.3 cm.), but it hugs the axis until about .31, after which it sweeps upward far beyond the chart into the hundred thousands and millions. The position of Wilson's negative ions and positive ions are indicated at .27 and above .31. Wilson's fog point would lie at .365 in the chart and there would be an air curve to the right beyond.

Weak Ionization. — If a weak ionizer (radium 10,000 X, 10 mg., sealed in an aluminum tube) is placed at $D = 40$ cm. from the glass fog chamber, the curve rises slightly above .24, becomes nearly constant slightly below .26, until above .29, after which it also begins to sweep with great rapidity into the hundred thousands of nuclei. In other words at weak ionization, the vapor nuclei of dust-free wet air become efficient in the presence of the ions. There are but two steps in the curve the initial one scarcely leaving the axis, the other at a height of about $n = 15{,}000$.

Moderate Ionization. — Let the radium tube be attached to the outer surface of the fog chamber. The curve obtained begins appreciably slightly above .22 ($v_1/v = 1.19$, adiabatic drop from 76 cm. about 18.4 cm.) but it scarcely rises until above .24. From this point it also sweeps

upward, but can not get much above 80,000 nuclei per cubic centimeter which condition is reached at about .27. To make this curve rise into the hundred thousands, *i. e.*, to make the vapor nuclei of dust-free wet air efficient in the presence of the ions, the exhaustion must be carried to about 50, much beyond the lateral limits of the diagram; but the fog is then intense and without coronas. No other apparatus has heretofore

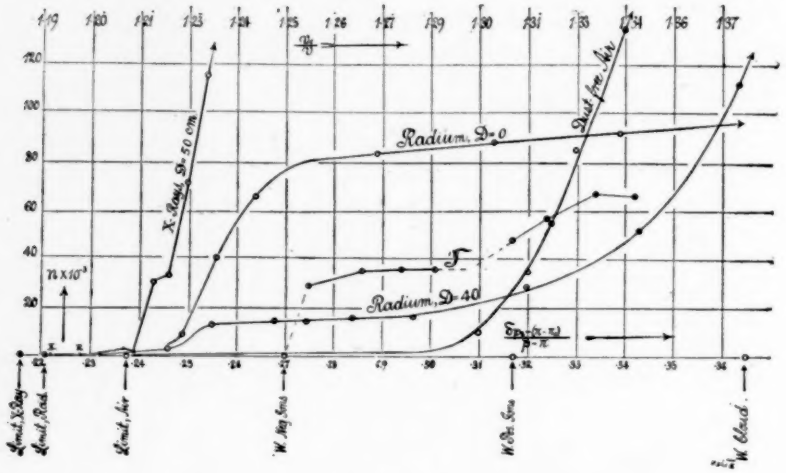

Fig. 1.

succeeded in catching the nuclei of dust-free air in the presence of such ionization. Again there are but two steps, one very near the axis not appreciably influenced by the greater ionization and the other above $n = 80{,}000$. Persistent nuclei are not produced, however long the exposure.

Strong Radiation. — If an ordinary X-ray bulb (4 inch spark) is placed at a distance of about 50 cm. from the fog chamber, the condensation produced begins appreciably somewhat below. 22 ($v_1/v = 1.18$; adiabatic drop from 76 cm. about 18 cm.); but the graph scarcely rises until nearly .23, when the upward sweep into the hundred thousands begins. Exposure of a few seconds produces fleeting nuclei only; exposure of one or more minutes produces persistent nuclei. In spite of intense ionization, the first step (.22 to .24) near the axis has scarcely risen, the other is indefinitely high beyond the reach of coronas.

Other Nucleations. — I have ventured to place J. J. Thomson's data (Phil. Mag., vol. V, 1903, p. 349) in the same chart at *T*. They must be interpreted, however, relatively to Wilson's points (negative ions $v_1/v = 1.25$, positive ions 1.31, cloud 1.38). In relation to the other curves of the chart Thomson's graph must be shifted bodily toward the left

until the lower and upper steps of the curve correspond with the other cases. In none of the experiments made with my apparatus does the initial step (which should correspond to Thomson's branch for negative ions), rise much above the horizontal axis, no matter how intense the ionization. This rise begins at about .24 in the chart and continues thereafter in a way to correspond with the ionization.

The diagram is too small vertically to include Thomson's second group of experiments in which the initial steps ($v_1/v < 1.33$) lies at an average height of $n = 85{,}000$.

Conclusion.—The ratio of one half in the height of the steps occurring in the region of transition from negative to positive ions, the accelerated motion of the piston producing exhaustion, and the agreement in size of predicted and observed ions, is the strong evidence of Thomson's interpretation. Nevertheless I am not prepared to abandon my own apparatus. H. A. Wilson with his method obtains a division of ions into three or more groups. What I get is another division of the totality of ions into two groups, one a numerically small group with large nuclei, and a numerically large group with relatively small nuclei and containing all the ions. This occurs even in such cases where I catch the vapor nuclei of dust free air in presence of the ions (radium at $D = 40$ cm.), and hence all ions, positive and negative, must have been caught in an earlier stage of the exhaustion.

The slopes of the air graph and the strong X-ray graph represent the initial branches of a general law of distribution of molecular aggregates, such as is given by the theory of dissociation. They may therefore be expected to be similar in their slopes as they actually are.

The question is finally to be asked why I catch the negative ions, etc., at an apparently much lower supersaturation than C. T. R. Wilson. I have entertained doubts whether the inertia of the piston in his apparatus is initially quite negligible; whether in any apparatus the computed adiabatic temperatures were actually reached. Nobody has proved it, and the case is worst for tubes. Moreover in every apparatus there must be a limit at which the smaller nuclei of a graded system can no longer be caught in the presence of the larger nuclei. But I do not believe that the real discrepancy will be found in any of these misgivings. It seems to me to be inherent in this; in Wilson's apparatus the results are given from the observed volume ratio v_1/v of adiabatic expansion; in my method the results follow from the observed pressure ratio p/p_1. It seems questionable whether the customary constants by which one passes from one group of data to the other are really applicable, to wet air at very low temperatures. Moreover when the exhaust cock in my apparatus is opened for

$t = 0$ .25 .5 1 2.5 5 ∞ sec.,

the isothermal pressures of the fog chamber (caet. par.) read

$\bar{p}_2 = (57.8)$ 53.2 52.7 52.0 51.5 50.9 50.4 cm.,

so that in the first quarter second of opening the final isothermal pressure p_3 (chambers communicating) is already reached to more than 60 per cent., and my smallest fog chamber holds over 6 litres. One naturally asks whether the importance of this in its bearing on the measurements of the ratio of specific heats k/c, has ever been adequately appreciated.

BROWN UNIVERSITY,
PROVIDENCE, R. I.

ON THE VARIATION OF THE HEAT OF MIXTURE WITH CONCENTRATION AND TEMPERATURE.[1]

BY B. M. CLARK.

WHEN two liquids are mixed, there is in general either an absorption or a generation of heat, the amount of heat depending upon the relative amounts of the liquids present, and upon the temperature at which mixture occurs.

The variation of the heat of mixture with temperature — for any definite concentration — bears theoretically a simple relation to the specific heats of the components of the mixture and the specific heat of the resultant mixture, and the present experiments were begun with a view to determining experimentally the numerical value of this relation in the case of some of the simpler solutions.

If c and c' represent the specific heats of the components of the solution, then the specific heat K of the solution will not in general be given by the simple law

$$K = c(1 - x) + c'x,$$

but will be greater or less than this value. Calling Q the heat of solution due to the mixture of any two liquids, we have from thermodynamic considerations the following relation

$$-\frac{d(Q_{xt})}{dt} = K - K'.$$

[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

When K is the observed specific heat of the resultant solution, K' the specific heat calculated from the above formula. This shows that only where $K = K'$ is the heat of solution independent of the temperature at which mixture occurs.

Observations were made with infinitely soluble as well as partially soluble liquids.

The values obtained for the heat of solution at various temperatures show that for the liquids tested the thermodynamic relation

$$-\frac{d(Q_{xt})}{dt} = K - K',$$

holds within the limits of experimental error. The value $\partial Q/\partial t$, known as the temperature coefficient of the heat of mixtures, varies in some mixtures with the temperature.

Anilin xylol, anilin-toluol gave evidence of a temperature coefficient equal to zero, *i. e.*,

$$\frac{\partial Q}{\partial t} = 0.$$

Numerical values and curves will be given in a paper to appear shortly.

The Constants in Gas-Viscosity.[1]

By Willard J. Fisher.

In 1893 W. Sutherland[2] proposed an explanation, based on the assumptions of the Kinetic Theory, with the additional one of attractive forces between molecules, which has been shown by numerous investigators to agree better with the observed facts of gas-viscosity than any of the empirical formulas. His formula is

$$\eta \propto \frac{\theta^{\frac{1}{2}}}{1 + \frac{C}{\theta}},$$

in which η is the viscosity-coefficient, θ the absolute temperature, C a constant depending on the gas. The formula holds only for temperatures above the critical and pressures such that Boyle's Law is true. Taking

[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

[2] W. Sutherland, Phil. Mag. (5), 36, p. 507, 1893.

η_0, the viscosity-coefficient at 0° C., as a second constant of the equation, all writers hitherto have thrown it into the form

$$\eta = \eta_0 \left(\frac{\theta}{273}\right)^{\frac{1}{2}} \frac{1 + \frac{C}{273}}{1 + \frac{C}{\theta}},$$

determining η_0 and C by the method of Least Squares.

Observing that in Sutherland's original article, in terms of molecular quantities, C has the value

$$\frac{1}{c} m^2 f\left(\frac{1}{2a}\right),$$

where m is the mass of a molecule, a its radius, $c/2$ the factor converting absolute temperatures into molecular kinetic energy, and f symbolizes the unknown law of potential energy between molecules, the writer plotted a curve between C and m^2, using the relative molecular weights of Chemistry; and finds that the elements which have been studied through sufficiently wide ranges of temperature give an approximately straight line, with equation $C = 0.058M^2 + 74$; in which M is the relative molecular weight.

Combining this with Sutherland's expression for C, for these gases,

$$f\left(\frac{1}{2a}\right) = \frac{c}{h^2}\left(0.058 + \frac{74}{M^2}\right),$$

in which h is the mass in grams of a Hydrogen atom. It is to be noted that the values of C are in considerable disagreement, as given by the different authors.

Also, instead of writing the equation with the variation sign, it may be put in the form

$$\eta = K \frac{\theta^{\frac{1}{2}}}{1 + \frac{C}{\theta}}$$

The proportionality constant K, hitherto unnoticed, has the value by Sutherland's article

$$\frac{.064 c^{\frac{1}{2}} m^{\frac{1}{2}}}{(2a)^2}$$

which suggests that K should be linear in $m^{\frac{1}{2}}$. Determining K from published data for various elements and compounds by a graphical method, it is found that no obvious law holds for the compounds, but that the four elements, H, N, O, Ar, give a straight line of formula $K \cdot 10^7 = 0.593M^{\frac{1}{2}} + 66$, in which M is the relative molecular weight.

Helium, in this respect, as in several others, does not form part of the series, and Argon deviates about 4 per cent.

No other elementary gases have been studied under conditions such as to justify farther conclusions.

Remarkable Optical Properties of Carborundum.[1]

By Lewis E. Jewell.

SOME transparent blue plates of carborundum, found in 1904, gave by preliminary measures with a microscope a very high value for the refractive index.

Later, in material received from the Carborundum Company of Niagara Falls, were found some transparent and colorless plates, some of which were fairly thick. The polarization of the plates was studied with the polariscope and one specimen of twinned crystals produced a well-defined, transparent, colorless prism where the plates were joined.

Measurements with the spectrometer were made and the refraction proved to be greater than that of the diamond, and the dispersion more than twice as great.

Spectrograms were obtained of the light transmitted through the plates and a thickness of one thirtieth of a millimeter absorbed practically all light having a wave length less than 4,000 Angström units.

Interference bands in the spectrum of light transmitted through thin parallel plates gave greater values of the dispersion than the measurements with the spectrometer. Spectrograms have also been obtained of the light reflected from the surfaces of the crystals and the reflective power, which is extremely high, differs very little in the visible and ultra-violet portions of the spectrum until a wave-length of about 2,400 is reached, beyond which there seems to be an absorption band.

There may be a slight increase in the reflective power in the violet and ultra-violet, but it is not very pronounced. After the measurements were made some better material was obtained and larger and much more perfect prisms have been made, with which much more accurate values of refractive indices are to be made, also the interference fringes in the spectra produced by carefully selected plates are to be used for this purpose also.

Very remarkable markings have been found upon some plates, the most remarkable of which take the form of spirals, some of which are hexagonal and others triangular, changing to hexagonal farther out. Most of

[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

the spirals are however circular, some of them are so close together and so regular as to give brilliant spectra, even to the sixth or eighth order; the lines in some cases are as close together as thirty or fifty thousand to the inch. Many of these markings are extremely beautiful.

Sometimes there are two outer coatings to the plates, the outer an oxidation product giving the brilliant surface colors which are due to a thin film. Sometimes there is another layer with an exceptionally brilliant silver-like surface. Either or both of these coatings may be present and in some cases neither one.

The first mentioned coating is comparatively soft and easily removed, the other coating is nearly or quite as hard as the corborundum itself.

THE MUTUAL INDUCTANCE OF A CIRCLE AND A COAXIAL HELIX. THE LORENZ EXPERIMENT AND THE AYRTON-JONES ABSOLUTE ELECTRODYNAMOMETER.[1]

BY E. B. ROSA.

THE constant of the Lorenz apparatus for the absolute measurement of resistance and of the Ayrton-Jones electrodynamometer for the absolute measurement of current require the calculation of the mutual inductance of a circle and a coaxial helix, the circle in the first case being the edge of the rotating disc and in the second case one end of the suspended coil, the helix being the fixed coil carrying the primary current.

Jones obtained an expression for the mutual inductance in question by integrating spirally along the entire length of the helix, his result being an exact expression but appearing in terms of elliptic integrals of all three kinds and hence being a very tedious and difficult expression to use in numerical calculations. Lorenz's expression is an algebraic series less difficult but also less accurate than Jones's. Using an expansion in zonal harmonics, I have obtained an expression in the form of an algebraic series, similar to Lorenz's but more accurate, which is far more convenient to use than Jones's and agreeing with the latter to less than one part in a million (in a particular numerical test) thus being amply accurate for the most refined experimental work. This expression is obtained on the assumption that the current is distributed over the solenoid in a uniform current sheet, whereas Jones's expression assumed the current flowing in a helix. Their agreement is a confirmation of the theorem that a spiral distribution of current is equivalent to a current sheet. For a short coil, however, a question arises as to how to introduce the current into the helix so as to meet the mathematical conditions of equivalence

[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

to a current sheet. It is shown in the paper that for such sizes of wire and coils as are likely to be employed in measurements of precision the variation in the constant of the coil due to the lead wires not conforming to the mathematical conditions may be made wholly inappreciable, so that the formula of Jones or the more convenient series formula may be used without any correction for leads.

Mutual Inductances for Laboratory Use.[1]

By E. B. Rosa.

THE most accurate as well as the most convenient method of calibrating a ballistic galvanometer is by means of a mutual inductance, through the primary of which a measured currrent flows. Some laboratories use a standard solenoid with a secondary wound within or without, calculating the mutual inductance of the primary and secondary coils from their dimensions. Such a solenoid made large enough and carefully enough to give the mutual inductance with fair precision is both bulky and expensive, and not very portable. Being of considerable length, its magnetic field often extends to a considerable distance and may disturb other work. On the other hand suitable primary and secondary coils may be wound on a thoroughly seasoned wood spool (or a marble spool) only 10 or 12 cm. in diameter and afford, when calibrated, an accurate and convenient mutual inductance for the calibration of galvanometers that is cheap, portable and reliable. Such inductances are so inexpensive and occupy so little space that one can be left with its secondary in circuit with the galvanometer, ready to be used for calibrating the latter at any time. They can be made in the laboratory at trifling cost and can be sent by mail or express to the Bureau of Standards and calibrated at very slight expense. Specifications for such coils will shortly be published by the bureau.

The Mutual Inductance of Two Coaxial Coils.[1]

By E. B. Rosa.

ABSOLUTE measurements of resistance have been made by Rowland, Glazebrook and others using two coaxial coils the mutual inductance of which was computed from their dimensions. By the methods employed by Kohlrausch and Rayleigh the mean radius of such coils can be obtained with great precision by comparing them with larger coils wound with a single layer of wire, the mean radius of which may be determined with sufficient accuracy by direct measurement. The formulæ

[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

employed by Rowland and Glazebrook in calculating the mutual inductances of their coils are both approximate, and do not agree with one another closely unless the coils have very small sections or are rather far apart. Weinstein and Stefan both gave formulæ for use in calculating the mutual inductance of such coils, but they do not agree with one another as accurately as is desirable for precision work. I have revised and corrected Weinstein's formula and derived a new formula starting where Stefan began, these two formulæ agreeing very closely and proving more accurate than any of the others. These formulæ are sufficiently accurate for the most refined work in the absolute measurement of resistance.

MODELS ILLUSTRATING THE MOTION OF A VIOLIN STRING.[1]

BY HARVEY N. DAVIS.

THE function $u(x, t)$ which gives the displacement, at the time t, of a point x units from one end of a violin string, can be represented graphically by a surface with the x, t plane as a base-plane. If the units were properly chosen, this graph would be the surface which the string would generate if, while it vibrated, it were also carried along in a direction perpendicular to the plane of its vibration; and in any case, a section of such a surface parallel to the x axis represents, usually on a magnified scale, the configuration of the string at some corresponding time, $t = t_0$, while one parallel to the t axis represents the displacement of a corresponding point, $x = x_0$, as a function of the time.

Five surfaces of this kind, modeled in three dimensions, were shown, representing, one the general Helmholtzian solution and the others the motion of a string bowed at points $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{7}$ and $\frac{3}{8}$ of its length from one end. These particular cases were chosen because, besides being typical, they have some interest in connection with Young's and Krigar-Menzel's observations on the absence or dominance of certain overtones when a string is bowed at or near one of their nodes.

THE MOTION OF A VIOLIN STRING UNDER LIGHT BOWING.[1]

BY HARVEY N. DAVIS.

THIS paper discusses the influence which the pressure of the bow upon the string has on the resulting vibration form.[2] For each bowing speed there is, for comparatively great pressures, a considerable range within which the only effect of a change in the pressure is a slight

[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

[2] The vibrations actually observed were longitudinal rather than transverse, the apparatus used being a modification of that described to the Society in December, 1905.

corresponding change in the position of equilibrium about which the string vibrates, the vibration form being always that described by Helmholtz and others, and the amplitude remaining the same. For smaller pressures both the amplitude and the vibration form change with the pressure. In particular, if the pressure is below a certain critical value, determined partly by the materials and condition of the apparatus and partly by the bowing speed, no vibrations can be maintained. For pressures slightly greater than this critical pressure there are no overtones, the time graph of the displacement of the point under the bow being a sine curve tangent at its point of greatest slope to the line representing the speed of the bow. As the pressure is increased beyond this value, the bowing speed remaining constant, the mode of vibration goes over continuously into the Helmholtzian form.

On Magnetic Shielding.[1]

By A. P. Wills.

ASSUMING the results of a previous paper, giving the "shielding ratio" for a set of three concentric spherical iron shells and for a similar set of cylindrical shells, the following problem was discussed: Given the innermost and outermost radii of the system in each case, what values should the remaining four radii have in order that the shielding shall be a maximum? Starting with the expression giving the "shielding ratio" (the ratio of field impressed to field within innermost shell), derived in the paper referred to above, the conditions for a maximum of the expression, under the conditions imposed, are examined; and it is found that approximately the best conditions are obtained when the radii of the shells are in geometrical progression. This holds for both spherical and cylindrical systems.

A Color Mixer.[1]

By Albert B. Porter.

THE simplest means of mixing and matching colors is undoubtedly Maxwell's color-top. It is, however, defective in that one cannot continuously vary the positions of the slotted discs while seeking a color-match, but must usually stop the top many times, readjust the discs, and again set up the rotation, before a satisfactory match is secured.

In the present instrument the slotted discs remain stationary while their image is rapidly rotated. This is effected by viewing the discs through a right-angled prism which rotated with its hypothenuse parallel to the axis of rotation and to the line of sight. As the slotted

[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

discs remain stationary, their angular exposure can be easily and continuously varied until the best possible color-match is secured.

ON THE NATURE OF OPTICAL IMAGES.[1]

BY ALBERT B. PORTER.

A CONSIDERATION of the method by which light is propagated is sufficient to show that an ordinary optical image is merely a particular case of an interference pattern. This may be easily shown experimentally by using as an object a coarse, black-line grating illuminated by a parallel beam of monochromatic light passing through the grating and then through a convex lens. On the far side of the lens a system of sharply defined interference fringes is formed which can be seen with an eyepiece, or intercepted on a screen, at any point over a considerable range along the axis. Somewhere in this system of fringes is the geometrical image of the grating, but it is visually quite indistinguishable from other sections of the fringe system. If the angle of incidence of the light falling on the grating is changed, the whole fringe system shifts to one side or the other except in the focal plane, where it remains stationary. This shows that the focal plane of the lens is merely the plane in which the interference fringes formed by light of all incidences coincide, and that the so-called geometrical image is really a superposition of coincident interference patterns; while the usual absence of a sharp image outside the focal plane is due to the more or less uniform illumination caused by the overlapping of fringe systems formed by light coming from various points in the source. When the grating is illuminated by a parallel beam of white light the effects are similar, except that outside the focal plane the interference fringes are colored. This shows that the focal plane is also the plane of achromatic interference, *i. e.*, the plane in which the fringes due to light of various wave-lengths coincide.

These experiments clearly show why it is in general essential to use a condenser to illuminate the field of a microscope in order to obtain a critical image, *i. e.*, an image which comes sharply into and out of focus and which is hence as free as possible from confusion with details of structure lying above and below the focal plane.

In the case of self-luminous objects, although the geometrical image is still to be considered as an interference pattern, the effects outside the focal plane are greatly modified by the lack of phase relations between the waves given off from various points of the object.

[1] Abstract of a paper presented at the New York meeting of the Physical Society, held December 27–29, 1906.

The Absorption of Some Solids for Light of Short Wave-length.[1]

By Theodore Lyman.

THE paper deals with light of extremely high frequency whose existence was first discovered by Schumann. This region of the spectrum presents problems of considerable interest, for to those questions of pure spectroscopy — the radiation and absorption of gases and solids and the reflection of metals — must be added problems connected with the photo-electric effect and with the physiological action of light.

The grating spectroscope which the author has used in his measurement of the hydrogen spectrum between 1650 and 1250 Angström units, though well adapted for the determination of wave-lengths is not especially fitted for rapid work in other branches of the subject. A prism spectroscope would be preferable for many purposes on account of its short light path and the superior intensity of its spectrum.

The object, therefore, of the present research was a purely practical one, namely, to find some substance from which the prism and lenses of such an instrument might be constructed. White fluorite is the only known substance fitted for the purpose, and it was hoped that a less costly substitute might be discovered.

In order to test rapidly the transparency of a great number of substances an instrument of special type was constructed. In it the light from a discharge tube fell upon a concave mirror and was thrown by it through a fluorite prism onto a screen coated with willimite. The whole apparatus was enclosed in an air-tight case which could be exhausted and filled with hydrogen. The specimens to be examined were introduced into this case in such a manner that eight of them could be interposed in succession between the mirror and source of light without opening the apparatus. The extent and intensity of the spectrum on the fluorescent screen was observed through a glass window.

To check the results obtained with this instrument the absorption spectra of some of these substances were photographed by a method already described.[2]

The substances examined were: Colored fluorites, quartz, topaz, gypsum, celestite, rock salt, barite, alum, colemanite, sugar (rock candy), borax, adularia, calcite, chrysoberyl, sanidin, anagonite, apophyllite, diamond and kunzite. The results are:

I. No substance shows so great transparency as white fluorite.

[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

[2] Astrophysical Journal, Vol. XXIII., No. 3.

II. The transparency of colored fluorites varies through a considerable range, but some light green specimens are nearly as good as the white variety.

III. Colored fluorites may be deprived of their color by heating. The process does not materially alter their transparency.

IV. Quartz in thicknesses of about 1 mm. is transparent to λ 1500. The absorption increases with such rapidity with the thickness that thick prisms and lenses would cut the spectrum off at about λ 1750 for all practical purposes.

V. Of the remaining substances Ceylon topaz is the best, being transparent to λ 1560.

VI. Rock salt is transparent to λ 1750 only.

ON THE CONDUCTIVITY OF THE AIR CAUSED BY CERTAIN COMPOUNDS DURING TEMPERTURE-CHANGE.[1]

BY FANNY COOK GATES.

INVESTIGATIONS on the conductivity of the air caused by the presence of the sulphate of quinine under certain conditions, indicate that it accompanies hydration and dehydration, although phosphorescent effects to which it may be directly due, appear at the same time. The phenomenon is most easily observed during the cooling of the quinine from something over 100° C. to room temperature.

Heretofore, the only other substance known to produce a similar effect is cinchonine, which like quinine, hydrates and phosphoresces upon cooling. Both substances are so complex in structure, that the exact chemical changes to which they are subject are studied with difficulty.

During the investigations described in the present paper, a large number of substances were heated to about 150° C., and the conductivity of the surrounding gas was tested while they cooled. Of all the substances tested, none was found to give so large an effect as that which results from the sulphate of quinine, but a very definite and marked conductivity was found to accompany the cooling of anthracene and to a less degree that of grape-sugar and æsculin.

THE TRANSMISSION OF RÖNTGEN RAYS THROUGH METALLIC SHEETS.[1]

BY J. M. ADAMS.

FOR some years after Röntgen's discovery of the X-rays, it was a matter of doubt whether the absorption of the rays in metallic sheets was accompanied by the development of heat. To investigate this question by means of an instrument different from those already

[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

used for this purpose, a radiomicrometer was constructed. The metals forming the thermal couple were constantan and copper. At one of the junctions was placed a small disk of thin platinum to receive the rays, the other junction being shielded from them. The sensitiveness of this radiomicrometer was such that a radiation of 5.6×10^{-8} gm. cal. per sec. per sq. cm. of its sensitive surface produced a deflection of one scale division.

Evidence of heat developed in the platinum by the absorption of Röntgen rays in it was readily obtained, and a quantitative measurement of the whole energy radiated per second in the form of Röntgen rays by a tube was made and was found to be of the same order of magnitude as the results of those investigators who had been able to observe the heating effect at all.[1] The necessity of making correction for the incomplete absorption of the rays in the platinum of the instrument, together with the well known fact that the character of Röntgen rays is changed by passage through substances, made it seem desirable to investigate the phenomena of the transmission of the rays through metallic sheets more fully than has been done heretofore.

The general law of the absorption of the rays in a metal, viz., that each successive equal increment of thickness is less effective as an absorbing medium than the one preceding it, was confirmed by experiments with the radiomicrometer; and curves showing the relation between the thickness of a metallic sheet and its absorbing power were plotted and were found to have the same general characteristics as similar curves which other investigators had obtained by means of the fluoroscope, the photographic plate, or the ionization electroscope.

The dependence of the absorbing power of a given metallic sheet upon the intensity of the rays incident upon it was examined for sheets of silver, platinum, copper, tin, and aluminium, and in every case it was found that the effectiveness of a sheet as an absorbing medium is independent of the intensity of the incident rays; in other words, the deflection of the radiomicrometer suffered an equal percentage reduction upon the interposition of a metallic sheet in the path of the rays, whether the latter were strong or weak. The rays were weakened by moving the tube away from the rest of the apparatus.

It was found that the effect of the *surfaces* of metallic sheets upon transmission is small, in the case of copper and of aluminium. To show this, a laminated plate of the metal in question was prepared, equal in total thickness to another solid plate of the metal. These two plates were interposed in turn in the path of the rays, and produced equal reductions of the deflection of the radiomicrometer.

[1] Several recent measurements by other investigators afford additional confirmation of the earlier results, so far as the order of magnitude is concerned.

The transmission of a beam of Röntgen rays through a metallic sheet has generally been supposed to render the beam more penetrating toward a second sheet of the same or of any other metal than the original beam was. Experiments with the radiomicrometer have confirmed this view in most cases; but the effect of transmission through silver on penetrating power for aluminium was found to be very small, while transmission through aluminium appeared to decrease the penetrating power of the rays for silver, so far as the deflections of the instrument are an indication.

Evidence of transformation of one sort of ray into another, in transmission through a metallic sheet, was sought, with a negative result. The experiment was performed by interposing in the path of the rays a plate made of two sheets of different metals placed face to face. If transformation occurs, the deflection of the instrument should depend upon the order in which the rays traverse the two sheets, for the effect of the second metal upon rays produced by the first would not be likely to be quantitatively the same as the effect of the first metal upon rays produced by the second. In fact, the order of the sheets in the plate made no difference in any case.

ON THE MAGNETIC PROPERTIES OF HEUSLER'S ALLOYS.[1]

BY J. C. MCLENNAN.

IN this paper the author describes some experiments made during the past year by Messrs. Dawes, McTaggart, and Robertson, and Miss L. B. Johnston on the magnetostriction and permeability of Heusler's alloys of varying composition.

From measurements on both phenomena the alloys are shown to be in an exceedingly unstable condition when freshly made and from observed changes in their magnetic behavior the conclusion is drawn that profound modifications are made in their structure through the lapse of time as well as by changes in temperature and by repeated magnetizations and demagnetizations.

In connection with the phenomenon of magnetostriction the gradual shortening observed by Austin[2] with rods of the alloys subjected to long continued high magnetic fields was shown to disappear when the rods reached a stable condition after being repeatedly magnetized.

In a series of rods containing the same amount of manganese with varying amounts of aluminium the greatest elongation was observed with rods in which the manganese and aluminium were present in the ratio of their atomic weights.

The paper concluded by emphasizing the suggestion made by Hill[3] that

[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

[2] PHYS. REV., 23, p. 498, 1906.

[3] Phil. Trans. A., CXCIV., 201, 1900.

possibly the magnetic properties of these alloys at room temperatures are largely determined by the temperatures from which they have been cooled, and that by suitably heating samples of the alloys to different temperatures and then chilling them their magnetic properties at these temperatures may be ascertained just as the structures of other alloys at different temperatures have been investigated in this way by Neville[1] and Heycock.

Geometrical Theory of Radiating Surfaces with Discussion of Light Tubes.[2]

By Edward P. Hyde.

THEORETICAL photometry assumes two general laws of radiation. (1) The law of variation of the intensity of illumination of a surface in inverse proportion to the square of the distance of the surface from the luminous source is merely a statement of a geometrical property, if the rectilinear propagation of light is assumed. (2) Lambert's law of variation of the intensity of a luminous surface in direct proportion to the cosine of the angle of emission is an empirical law based primarily on the observation that a uniformly bright sphere, when viewed at a distance appears as a uniformly bright disk. It would seem to follow from Kirchhoff's law that Lambert's cosine law can be true only for a black body, but no satisfactory experiments have been made so far as the writer knows, to test the law empirically in its application to glowing surfaces.

These two laws, the inverse square law and the cosine law, are applicable to the infinitesimal elements of a radiating surface, and large errors may result if they are assumed to apply to an extended source as a whole. Particularly is this so in the case of the inverse square law, which underlies the great majority of photometric measurements. In a previous paper[3] the writer solved the case for a finite cylinder of uniform brightness, in connection with a study of Talbot's law as applied to the rotating sectored disk. The curve of deviation from the inverse square law for a cylinder 20 mm. long and 1 mm. radius (the distances used in applying the inverse square law being measured to the center of the cylinder) is plotted, and is resolved into two simple curves corresponding to the two extreme cases of (1) a cylinder 20 mm. long but with an infinitesimal radius, and (2) a cylinder with a radius of 1 mm. but having an infinitesimal length.

Another simple illustration of the treatment of a radiating surface, as

[1] Proc. Roy. Soc., LXIX., 320, 1902.

[2] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906. To be published in full in the Bureau of Standards Bulletin.

[3] Bureau of Standards Bulletin, Vol. 2, p. 1, 1906.

distinguished from a point source, is had in the case of a uniformly bright circular disk of finite radius, for which the curve of deviation from the inverse square law is plotted.

In each of the above two cases the expression is deduced for the illumination at different distances in a single plane or along a single line symmetrical with respect to the radiating surface. In the case of a uniformly bright strip of infinite length but of finite width it is not difficult to derive the expression which will give the illumination at any point in space. Instead of illumination, however, the more general term *specific luminous flux* is substituted. At every point in space there is some definite direction in which the flux of luminous energy is a maximum. The quantity of luminous energy which in one second flows normally across a surface of unit area placed perpendicular to the direction of maximum flux is defined as the *specific luminous flux* at the point. It is a vector quantity, and the component in any direction equals numerically the difference in illumination on the two sides of an infinitely thin material screen placed perpendicular to the direction.

Since the problem of the infinite strip is one of two dimensions the complete solution is obtained from a consideration of a single plane normal to the strip and intersecting it in the line MN. Denoting the specific luminous flux by Φ_0 it is found that Φ_0 at any point is in the direction of the tangent at the point to the hyperbola having M and N as foci. If we draw the system of confocal hyperbolas and ellipses having M and N as foci, the hyperbolas intercepting equal distances on the strip MN, and if we remember that this system of hyperbolas and ellipses is a plane section of hyperbolic and elliptic cylinders, the hyperbolic cylinders of unit height may be considered as tubes of light, the specific luminous flux being along the tube at every point.

By deducing the flux of light across successive elliptic cylinders between two consecutive hyperbolic cylinders, it is found that the specific luminous flux is solenoidal, and therefore its divergence is zero. The elliptic cylinders may be considered as level surfaces, and a special form of potential function Ψ, may be derived such that along any *one* hyperbola Φ_0 is proportional to the rate of decrease of Ψ in the direction of the tangent at the point, At the intersection of different hyperbolas with any one ellipse, however, Φ_0 is proportional to the rate of decrease of Ψ in the direction of the normal to the ellipse, multiplied by $\sin y$, where $\sin y$ is the semi-minor axis of successive hyperbolas corresponding to different values of y. It readily follows that the second differential parameter $\Delta \Psi = 0$.

Since two examples have been given of the errors incident to assuming for a finite surface the inverse square law, which only applies to the elements of the surface, the above case of an infinite strip is used to show

the errors incident to assuming for the strip as a whole the cosine law which is true only for the elements of surface of the strip. The values of Φ_0 at different distances in a direction normal to the strip at its middle point, and in a direction making an angle of 45° with the normal at the middle point are calculated. For any definite distance the former multiplied by cos 45° would equal the latter if the cosine law, which has been assumed for the elements of surface, applied to the surface as a whole. The difference between the two values gives the errors for the distance used, and by plotting the errors for different distances a curve of deviation from the cosine law is obtained.

The Electrode Equilibrium of the Standard Cell.[1]

By F. A. Wolff and C. E. Waters.

SOME experiments recently described by Hulett[2] indicating the existence of a state of unstable equilibrium in the Weston cell have, on account of the importance of the subject, led to a further study of the question at the Bureau of Standards.

The equilibrium between cadmium amalgam and cadmium sulphate was first studied. Saturated solutions of the latter were shaken up in air, nitrogen, hydrogen and in vacuo with cadmium amalgam, but no difference as great as 10 microvolts was obtained when the treated solutions were employed in setting up Weston cells, even though the samples shaken in the presence of air were cloudy from the basic cadmium sulphate formed.

The equilibrium of mercury, mercurous sulphate and cadmium sulphate, and the corresponding system of the Clark cell was then studied in special cells, so constructed that the above materials could be rotated and the effect determined without opening the cells.

The cell consisted of a tube about 2 cm. in diameter and 12 cm. long, provided at the lower end with a small bulb into which a platinum wire was sealed. The bulb was connected to the main tube by a narrow neck, so that, with sufficient mercury in the cell, the platinum terminal was never in contact with the solution. A shorter internal tube about 1 cm. in diameter with several lateral openings, sealed in at the other end of the main tube, was charged with amalgam and crystals of cadmium or zinc sulphate which were held in place by a plug of asbestos wool. A platinum wire, sealed into a glass tube, to protect it from the solution extended from the amalgam through the upper seal, made after the introduction of the above materials.

The mercurous sulphate or cadmium sulphate or zinc sulphate crystals

[1] Abstract of paper read at the New York meeting of the American Physical Society, December, 1906.

[2] Phys. Rev., 23, 166, August, 1906.

were introduced over the mercury through a side tube by which the cell was next exhausted and through which a saturated solution of cadmium or zinc sulphate was subsequently introduced. After filling, the side tube was sealed off, leaving only a small air bubble in the cell, thus practically eliminating the influence of the air.

The cells were then placed in holders so arranged that one dozen could be rotated simultaneously with their axes parallel to the axis of rotation and inclined at a small angle to the horizontal.

Altogether 12 Weston and 5 Clark cells of this type were set up with samples of mercurous sulphate, made not only by the electrolytic method but also by several chemical methods.

The Weston cells after rotating from one to two months in a constant temperature oil-bath show changes which do not exceed 0.01 per cent. for all the gray samples (the grayness probably due to very finely divided mercury) except in one case in which the platinum terminal was purposely exposed to the solution.

The white samples generally showed effects several times as great as the gray ones, but not as large as found by Dr. Hulett. There also seems to be a tendency to reach a maximum, and then to approach normal values.

One Weston cell set up with basic cadmium sulphate has remained as constant as any of the others, its value being a few parts in 100,000 below normal.

The Clark cells showed effects of the same nature as the Weston, but in general the differences were even less, particularly for the white samples of mercurous sulphate.

No initial low values, as described by Hulett, were ever obtained.

The above results are given to show that *not all* samples of mercurous sulphate exhibit the behavior observed by Hulett and that the gray samples were, after a rotation of several months, still in agreement with cells of the usual construction to within 0.01 per cent.

This investigation is still under way and when completed the results, with full details, will be published in the Bulletin of the Bureau of Standards.

THE STANDARD CELL.[1]

BY FRANK A. WOLFF AND C. E. WATERS.

THE paper is a resumé of the work done by the authors, at the Bureau of Standards, on the reproducibility of the Clark and Weston standard cells.

Mercurous sulphate, the component to which most of the differences,

[1] Abstract of a paper presented at the New York meeting of the Physical Society, December 27–29, 1906.

in the past, may be attributed, was prepared by the following methods.

1. The electrolytic method, devised independently by Carhart and Hulett and one of the authors, in which the mercurous sulphate is formed by the passage of the current from a mercury anode, the electrolyte being sulphuric acid. Current densities from less than 0.1 to more than 9 amperes per square decimeter were employed while the strength of acid varied from 20 to 400 grams per liter. Two different forms of apparatus were used and the stirring was varied between wide limits.

2. By the recrystallization of commercial samples from strong sulphuric acid under various conditions (5 samples).

3. By the action of fuming sulphuric acid on metallic mercury (1 sample).

4. By treatment of mercurous nitrate with sulphuric acid (2 samples).

5. By reduction of mercuric sulphate by metallic mercury (2 samples).

6. By reduction of mercuric sulphate by SO_2 (2 samples).

7. By the direct action of rather strong sulphuric acid, containing a small percentage of nitric acid, on mercury (6 samples).

In addition six commercial samples were treated as follows:

8. By digestion with hot 1:4 sulphuric acid and mercury for several hours (6 samples).

9 and 10. By digestion in the cold with 1:4 sulphuric acid since April, 1904 (two lots of 5 and 4 samples respectively).

11. By digestion in the cold with 1:6 sulphuric acid and mercury since October, 1905 (6 samples).

In addition two samples of mercurous sulphate made by Professor Hulett were available.

Weston and Clark cells were set up for each of these samples, the remaining ingredients being:

Mercury redistilled at reduced pressure.

Cadmium — electrolytic, obtained from Kahlbaum and the same redistilled several times in vacuo.

Cadmium sulphate — two samples obtained from Kahlbaum. Recrystallized — cloudy crystals.

Zinc from Kahlbaum grade I. and the same redistilled in vacuo.

Zinc sulphate — samples from Kahlbaum and Baker recrystallized.

The Results.

Of 55 Weston cells set up with 38 samples of electrolytic mercurous sulphate, the deviation of 31 from the mean of all was within ± 10 microvolts, of 14 from 10 to 15 microvolts and of 5 from 15 to 20 microvolts. Of the remaining five (maximum deviation less than 50 microvolts) there were 4 duplicates of which the deviation did not exceed 11 microvolts.

Resumé of Results.

Mercurous Sulphate.	Number of Samples.	Weston Cells.		Clark Cells.	
		Number of Cells.	Average Deviation From Mean of Electrolytic Cells.	Number of Cells.	Average Deviation From Mean.
Method 1	38	55	±11 microvolts.	57	±10 microvolts.
2	5	5	−12 "	5	−10 "
3	1	1	−04 "	1	−02 "
4	2	4	−18 "	2	00 "
5	2	2	−12 "	2	−04 "
6	2	2	+18 "	2	+05 "
7	6	9	−14 "	10	−11 "
8	6	6	+ 4 "	8	+18 "
9	5	5	+27 "	5	37
10	4	4	+50 "	—	+— "
11	6	6	+70 "	8	+90 "
Hulet's I.	1	2	+30 "	3	+32 "
" II.	1	3	−01 "	3	+07 "

Of 57 Clark cells set up with the same samples of mercurous sulphate, 35 differed from the mean of all by less than 10 microvolts, 15 by 10 to 15 microvolts, 6 by 15 to 25 microvolts, while the remaining cell differed by 36 microvolts from the mean.

The results for all the samples of mercurous sulphate are given in the table above.

In addition 22 Weston cells were set up to determine what variations might be introduced from the cadmium sulphate, and whether there were any electromotive differences between clear and cloudy crystals as suggested by Prof. Hulett. Eight samples in all, recrystallized from acid, basic and neutral solutions, were tried and, excluding one sample precipitated at high temperature and no doubt inverted, the mean deviation from the electrolytic cells was found to be — 6 microvolts with a range of + 9 to — 29 microvolts.

The values obtained from day to day generally agree to within a few parts per million and although most of the Weston cells of the Bureau are seven months old no tendency toward a decrease in electromotive force as observed by Hulett has been found. A small number of cells which showed high initial values have decreased, but not below the mean of all the cells made with electrolytic mercurous sulphate. The same remark applies to samples made by the various chemical methods employed.

In addition, the ratio of Clark to Weston cells was determined from time to time, but the small differences observed, amounting to two or three parts in 100,000, and including errors of temperature measurement

and those due to defective insulation of the cells in the bath and between the individual coils of the high resistance potentiometer employed, indicate an apparent *increase* of the Weston cells, or a *decrease* of the Clark cells.

A number of Weston cells recently set up have somewhat *smaller* values than the older cells, prepared from the same materials, but the difference does not exceed 10 microvolts.

Clark and Weston cells have been exchanged with Professor Hulett and very recently with Professor Carhart and Professor Guthe. In addition four electrolytic Weston cells made at Cornell University by Mr. Wold have been compared at the Bureau. The results of these comparisons are given below in terms of the means of the Bureau cells.

	Weston Cells.		Clark Cells.	
	Cell.	Mean Difference in Microvolts.	Cell.	Mean Difference in Microvolts.
Prof. Hulett	A_3	+45	H_4	−15
" "	A_{32}	+27	H_{18}	+5
" "	O_4	40		
Prof. Carhart	No. 5	+1	C_2	00
" "	No. 8	+4		
Prof. Guthe	C_1	+25	A_1	+65
Mr. Wold	No. 24	−9		
" "	No. 27	−10		
" "	No. 20	−24		
" "	No. 21	−15		

In addition two other cells made by Professor Hulett have been under observation since July, 1906. F_7, one of the first electrolytic cells set up by him several years ago, has steadily decreased form — 10 to — 150 microvolts, while O_5 has shown great fluctuations, the value ranging from — 100 to + 32 to — 60.

While the direct comparison of cells made by Professor Hulett with those made at the Bureau shows a discrepancy of 25–45 microvolts, cells made by the anthors with sample II. of mercurous sulphate show that the differences are probably not due to the mercurous sulphate used but to some difference in the other materials or in the specifications employed.

Samples of the various materials will therefore be exchanged with other investigators in this field, to determine the cause of difference, and it is confidently expected that as excellent agreement as is here shown can be reached by all when the same specifications are followed.

Further work is in progress at the Bureau of Standards on the remaining cell materials and on other factors which might influence the electromotive force, *e. g.*, size of grain, depth of paste, influence of diffusion, influence of added impurities. A careful determination of the temperature coefficients will also be made.

The main conclusions to be drawn are that since samples of mercurous sulphate of uniform electromotive properties can be made by a number of different methods, and, since the remaining materials used in the construction of standard cells can be obtained in an exceedingly pure state, that Clark and Weston standard cells are reproducible to within two or three parts in 100,000, that they reach a permanent value within a few days, and that when set up with proper materials the Weston cell can be depended on as constant well within the above limits for a considerable period.

Another conclusion that might be drawn is that the standard cell has a marked superiority, both in ease and accuracy of reproduction over the silver coulometer as a *fundamental* electrical standard, particularly as the results obtainable with the latter can only be preserved by the aid of a standard current balance, or the combination of a standard resistance and a standard cell, all of which must be assumed constant between silver coulometer measurements.

With materials at hand, and if necessary, these could be procured by investigators from the various National Physical Laboratories, a number of cells can very readily be set up in far less time than that required for a coulometer measurement, and with a greater certainty of reproducing the volt than is possible by the coulometer and standard resistance, both of which are subject to error.

The choice of the standard cell and the standard resistance as fundamental standards is besides the most logical as practically all voltage and current measurements of precision are made by the potentiometer method and thus directly in terms of the standard cell or the combination of a standard cell and a standard resistance.

Considerable work has already been done along other lines and will, with the full details of this paper, be published an early number of the Bulletin of the Bureau of Standards.